The "geosphere" package (Version 1.2-4)

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1 Introduction

This vignette describes the R package 'geosphere'. The package implements spherical trigonometry functions for geographic applications. Many of the functions have applications in navigation, but others are more general, or have no relation to naviation at all.

There are a number of functions to compute distance and direction (= bearing, azimuth, course) along great circles (= shortest distance on a sphere, or "as the crow flies") and along rhumb lines (lines of constant bearing).

Other functions include the computation of the location of an object at a given direction and distance; and the area, perimiter, and centroid of a spherial polygon.

Geographic locations must be specified in longitude and latitude (and in that order!) in degrees (i.e., NOT in radians). Degrees are (obviously) in decimal notation. Thus 12 degrees, 30 minutes, 10 seconds = 12 + 10/60 + 30/3600 = 12.175 degrees. The southern and western hemispheres have a negative sign.

The default unit of distance is meter; but this can be adjusted by supplying a different radius 'r' to functions. Directions are expressed in degrees (N = 0 and 360, E = 90, S = 180, and W = 270 degrees). Arguments of functions that take several arguments (e.g. points, bearings, radius of the earth), are cbind-ed together.

The functions in this package are mostly based on formulae provided by Ed Williams (`http://williams.best.vwh.net/ftp/avsig/avform.txt`, and partly on javascript implementations of these formulae by Chris Veness (`http://www.movable-type.co.uk/scripts/latlong.html`)

This is one of the first versions of this package, so please look out for bugs and let us know if you find any.

2 Great circle distance

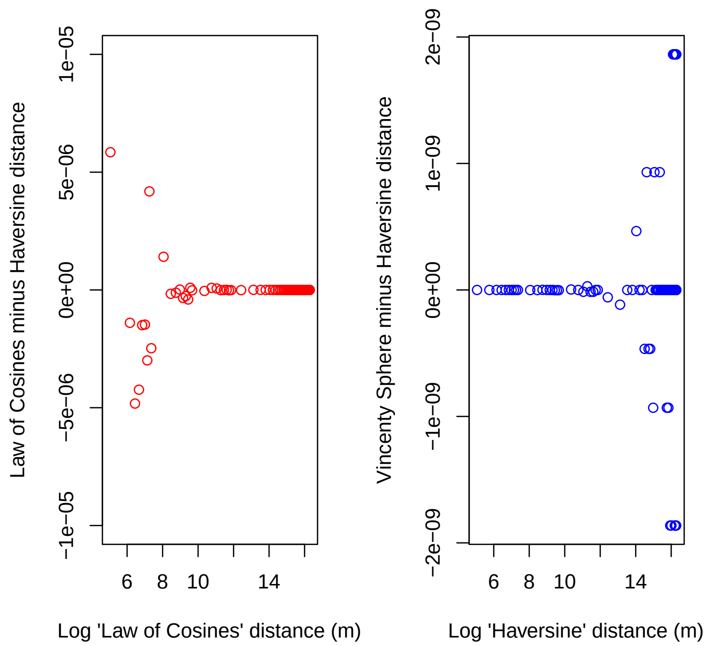
There are four different functions to compute distance between two points. These are, in order of increasing complexity of the algorithm, the 'Spherical law of cosines', 'Haversine' (Sinnott, 1984), 'Vincenty Sphere' and 'Vincenty

Ellipsoid' (Vincenty, 1975) methods. The first three assume the earth to be a sphere, while the 'Vincenty Ellipsoid' assumes it is an ellipsoid (which is closer to the truth).

The results from the first three functions are identical for practical purposes. The Haversine ('half-versed-sine') formula was published by R.W. Sinnott in 1984, although it has been known for much longer. At that time computational precision was lower than today (15 digits precision). With current precision, the spherical law of cosines formula appears to give equally good results down to very small distances. If you want greater accuracy, you could use the distVincentyEllipsoid method.

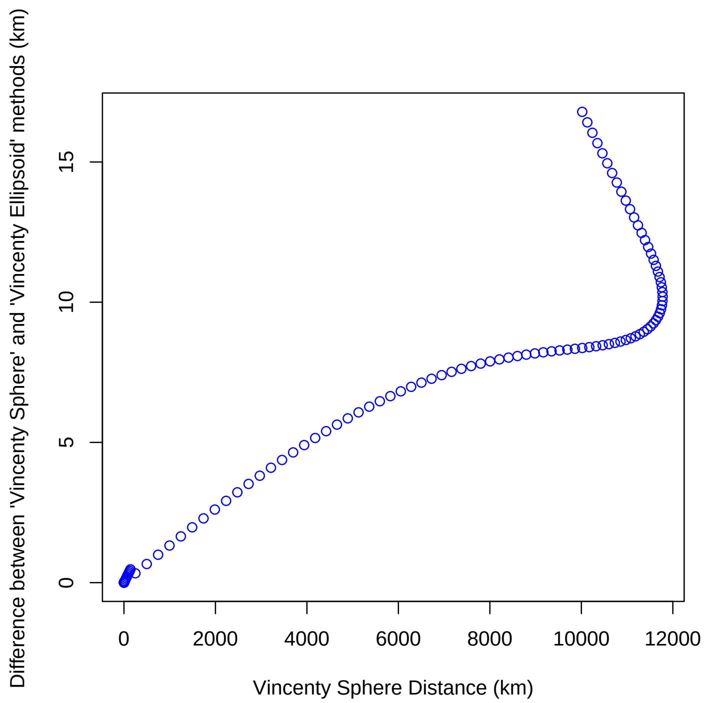
Below the differences between the three spherical methods are illustrated. At very short distances, there are small differences between the 'law of the Cosine' and the other two methods. There are even smaller differences between the 'Haversine' and 'Vincenty Sphere' methods at larger distances.

```
> library(geosphere)
> Lon = c(1:9/1000, 1:9/100, 1:9/10, 1:90*2)
> Lat = c(1:9/1000, 1:9/100, 1:9/10, 1:90)
> dcos = distCosine(c(0,0), cbind(Lon, Lat))
> dhav = distHaversine(c(0,0), cbind(Lon, Lat))
> dvsp = distVincentySphere(c(0,0), cbind(Lon, Lat))
> par(mfrow=(c(1,2)))
> plot(log(dcos), dcos-dhav, col='red', ylim=c(-1e-05, 1e-05),
+             xlab="Log 'Law of Cosines' distance (m)",
+             ylab="Law of Cosines minus Haversine distance")
> plot(log(dhav), dhav-dvsp, col='blue',
+             xlab="Log 'Haversine' distance (m)",
+             ylab="Vincenty Sphere minus Haversine distance")
```



The difference with the 'Vincenty Ellipsoid' method is more pronounced. In the example below (using the default WGS83 ellipsoid), the difference is about 0.3

```
> dvse = distVincentyEllipsoid(c(0,0), cbind(Lon, Lat))
> plot(dvsp/1000, (dvsp-dvse)/1000, col='blue', xlab='Vincenty Sphere Distance (km)',
+         ylab="Difference between 'Vincenty Sphere' and 'Vincenty Ellipsoid' methods (km)")
```



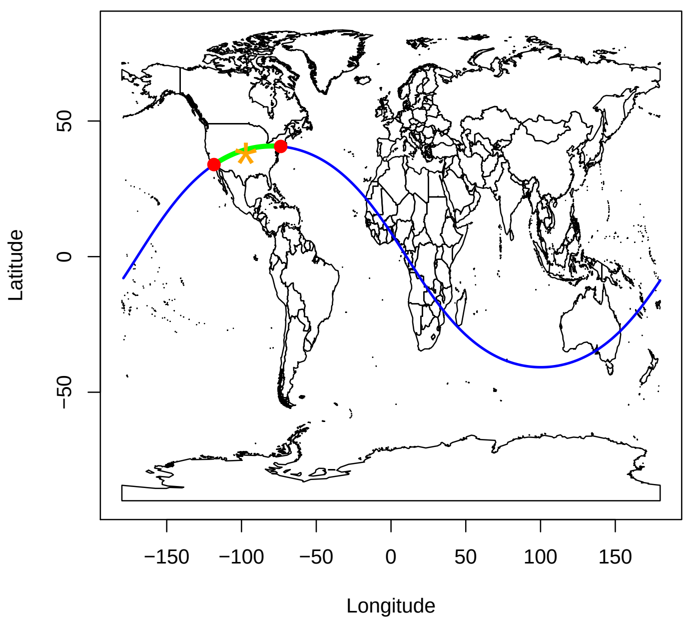
3 Points on great circles

Points on a great circle are returned by the function 'greatCircle', using two points on the great circle to define it, and an additional argument to indicate how many points should be returned. You can also use greatCircleBearing, and provide starting points and bearing as arguments. gcIntermediate only returns points on the great circle that are on the track of shortest distance between the two points defining the great circle; and midPoint computes the point half-way between the two points.

```
> LA <- c(-118.40, 33.95)
> NY <- c(-73.78,  40.63)
> data(wrld)
> plot(wrld, type='l')
> gc <- greatCircle(LA, NY)
> lines(gc, lwd=2, col='blue')
> gci <- gcIntermediate(LA, NY)
> lines(gci, lwd=4, col='green')
> points(rbind(LA, NY), col='red', pch=20, cex=2)
> mp <- midPoint(LA, NY)
> points(mp, pch='*', cex=3, col='orange')
```

```
> greatCircleBearing(LA, brng=270, n=10)
```

```
       lon        lat
 [1,] -144  31.264003
 [2,] -108  33.511699
 [3,]  -72  24.904460
 [4,]  -36   5.088211
 [5,]    0 -17.755535
 [6,]   36 -31.264003
 [7,]   72 -33.511699
 [8,]  108 -24.904460
 [9,]  144  -5.088211
[10,]  180  17.755535
```

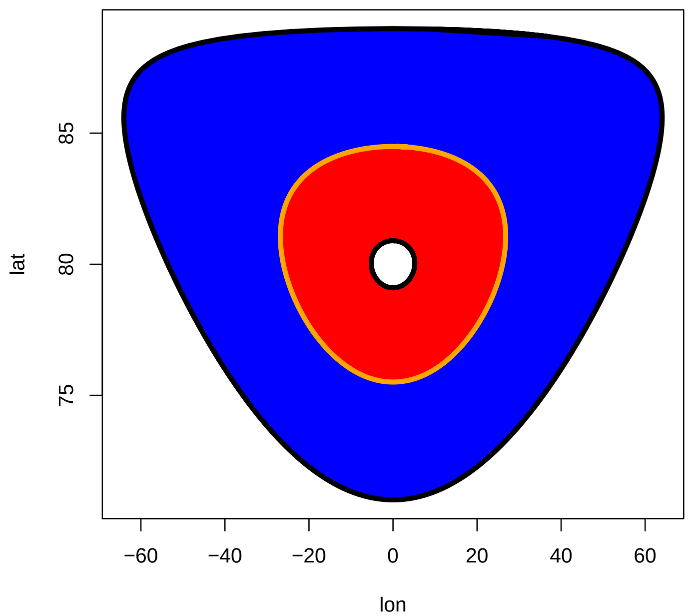


4 Point at distance and bearing

Function destPoint returns the location of point given a point of origin, and a distance and bearing. Its perhaps obvious use in georeferencing locations of distant sitings. It can also be used to make circular polygons (with a fixed radius, but in longitude/latitude coordinates)

```
> destPoint(LA, b=65, d=100000)
```

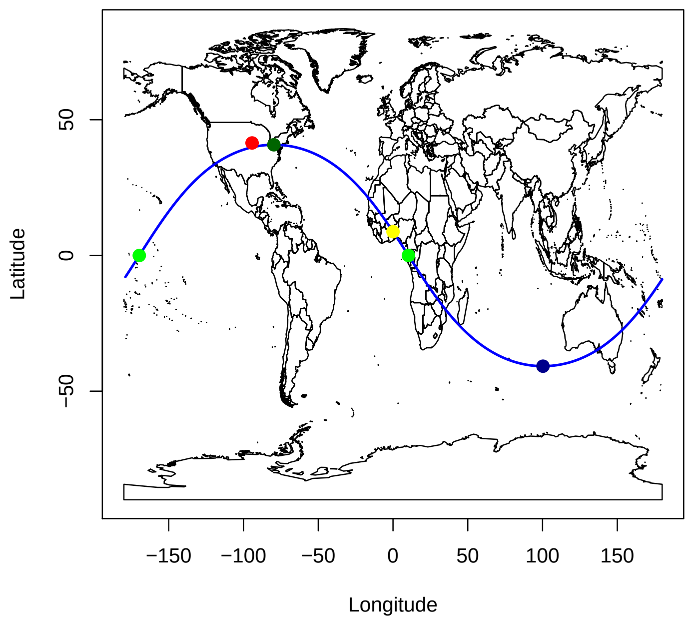
```
      lon      lat
 -117.4142 34.32572

> circle=destPoint(c(0,80), b=1:365, d=1000000)
> circle2=destPoint(c(0,80), b=1:365, d=500000)
> circle3=destPoint(c(0,80), b=1:365, d=100000)
> plot(circle, type='l')
> polygon(circle, col='blue', border='black', lwd=4)
> polygon(circle2, col='red', lwd=4, border='orange')
> polygon(circle3, col='white', lwd=4, border='black')
```



5 Maximum latitude on a great circle

You can use the functions illustrated below to find out what the maximum latitude is that a great circle will reach; at what latitude it crosses a specified longitude; or at what longitude it crosses a specified latitude. From the map below it appears that Clairaut's formula, used in gcMaxLat is not very accurate. Through optimization with function greatCircle, a more accurate value was found. The southern-most point is the antipode (a point at the opposite end of the world) of the northern-most point.

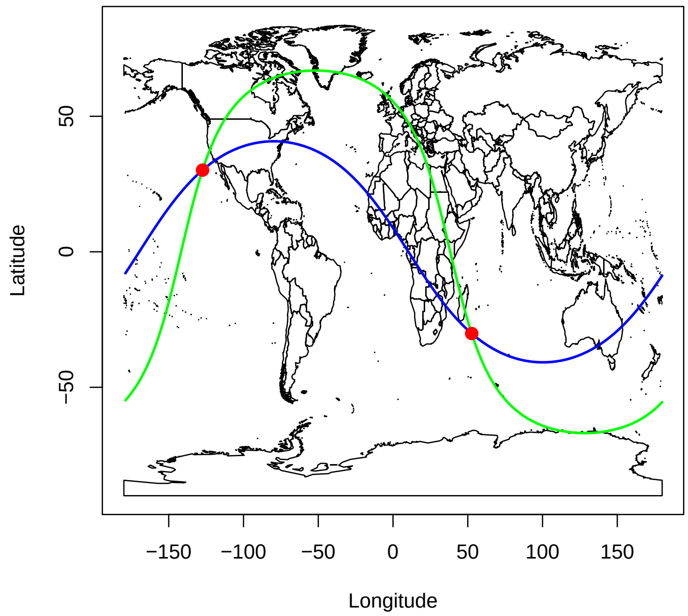
```
> ml <- gcMaxLat(LA, NY)
> lat0 <- gcLat(LA, NY, lon=0)
> lon0 <- gcLon(LA, NY, lat=0)
> plot(wrld, type='l')
> lines(gc, lwd=2, col='blue')
> points(ml, col='red', pch=20, cex=2)
> points(cbind(0, lat0), pch=20, cex=2, col='yellow')
> points(t(rbind(lon0, 0)), pch=20, cex=2, col='green' )
> f <- function(lon){gcLat(LA, NY, lon)}
> opt <- optimize(f, interval=c(-180, 180), maximum=TRUE)
> points(opt$maximum, opt$objective, pch=20, cex=2, col='dark green' )
> anti <- antipode(c(opt$maximum, opt$objective))
> points(anti, pch=20, cex=2, col='dark blue' )
```



6 Great circle intersections

Points of intersection of two great circles can be computed in two ways. We use a second great circle that connects San Francisco with Amsterdam. We first compute where they cross by defining the great circles using two points on it (gcIntersect). After that, we compute the same points using a start point and initial bearing (gcIntersectBearing). The two points where the great circles cross are antipodes. Antipodes are connected with an infinite number of great circles.

```
> SF <- c(-122.44, 37.74)
> AM <- c(4.75, 52.31)
> gc2 <- greatCircle(AM, SF)
> plot(wrld, type='l')
> lines(gc, lwd=2, col='blue')
> lines(gc2, lwd=2, col='green')
> int <- gcIntersect(LA, NY, SF, AM)
> int

         lon1      lat1      lon2     lat2
[1,] 52.62562 -30.15099 -127.3744 30.15099

> antipodal(int[,1:2], int[,3:4])

[1] TRUE

> points(rbind(int[,1:2], int[,3:4]), col='red', pch=20, cex=2)
> bearing1 <- bearing(LA, NY)
> bearing2 <- bearing(SF, AM)
> bearing1

[1] 65.89757

> bearing2

[1] 29.75541

> gcIntersectBearing(LA, bearing1, SF, bearing2)

          lon       lat       lon      lat
[1,] 52.62562 -30.15099 -127.3744 30.15099
```



7 Triangulation

Below is triangulation example. We have three locations (NY, LA, MS) and three directions (281, 60, 195) towards a target. Because we are on a sphere, there are two (antipodal) results. We only show one here (by only using int[,1:2]). We compute the centroid from the polygon defined with the three points. To accurately draw a spherical polygon, we can use makePoly. This function inserts intermediate points along the paths between the vertices proviced (default is one point every 10 km).

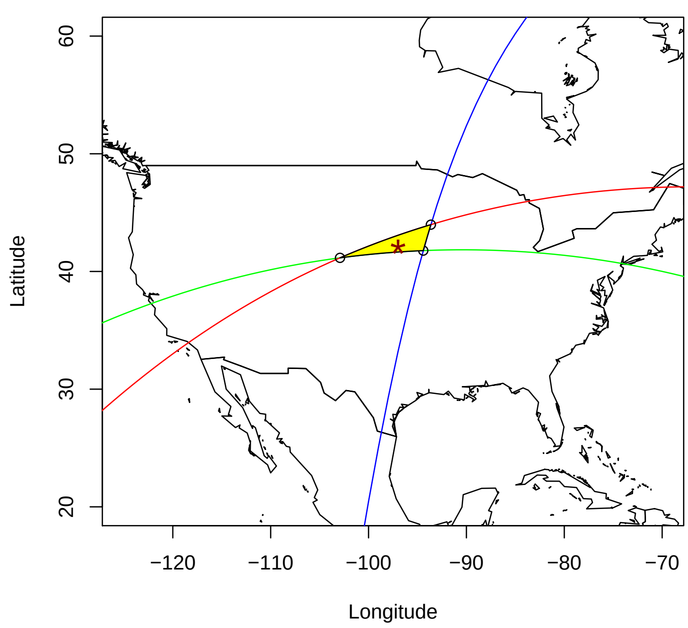
```
> MS <- c(-93.26, 44.98)
> gc1 <- greatCircleBearing(NY, 281)
> gc2 <- greatCircleBearing(MS, 195)
> gc3 <- greatCircleBearing(LA, 55)
> plot(wrld, type='l', xlim=c(-125, -70), ylim=c(20, 60))
> lines(gc1, col='green')
> lines(gc2, col='blue')
> lines(gc3, col='red')
> int <- gcIntersectBearing(rbind(NY, NY, MS),
+               c(281, 281, 195), rbind(MS, LA, LA), c(195, 55, 55))
> int
```

```
           lon      lat      lon       lat
[1,]  -94.40975 41.77229 85.59025 -41.77229
[2,] -102.91692 41.15816 77.08308 -41.15816
[3,]  -93.63298 43.97765 86.36702 -43.97765
```

```
> distm(rbind(int[,1:2], int[,3:4]))
```

```
           [,1]       [,2]     [,3]     [,4]     [,5] [,6]
[1,]        0.0         NA       NA       NA       NA   NA
[2,]   712641.5        0.0       NA       NA       NA   NA
[3,]   253542.4   822706.2        0       NA       NA   NA
[4,] 20037508.3 19324866.8 19783966      0.0       NA   NA
[5,] 19324866.8 20037508.3 19214802 712641.5      0.0   NA
[6,] 19783965.9 19214802.1 20037508 253542.4 822706.2    0
```

```
> int <- int[,1:2]
> points(int)
> poly <- rbind(int, int[1,])
> centr <- centroid(poly)
> poly2 <- makePoly(int)
> polygon(poly2, col='yellow')
> points(centr, pch='*', col='dark red', cex=2)
```



8 Bearing

Below we first compute the distance and bearing from Los Angeles (LA) to New York (NY). These are then used to compute the point from LA at that distance in that (initial) bearing (direction). Bearing changes continuously when traveling along a Great Circle. The final bearing, when approaching NY, is also given.

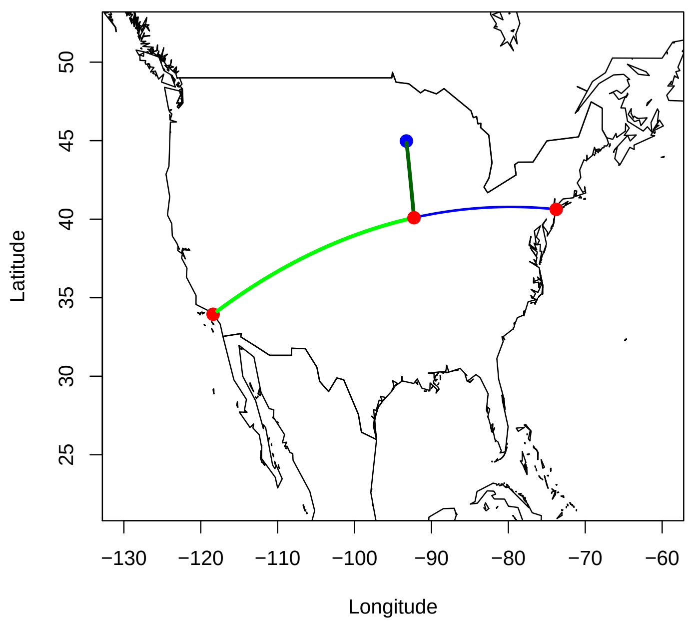
```
> d <- distCosine(LA, NY)
> d

     distance
[1,]  3977614

> b <- bearing(LA, NY)
> b

[1] 65.89757

> destPoint(LA, b, d)

    lon   lat
 -73.78 40.63

> NY

[1] -73.78  40.63

> finalBearing(LA, NY)

[1] 93.86472
```

9 Getting off-track

What if we went off-course and were flying over Minneapolis (MS)? The closest point on the planned route (p) can be computed with the alongTrackDistance and destPoint functions. The distance from 'p' to MS can be computed with the crossTrackDistance function. The light green line represents the along-track distance, and the dark green line represents the cross-track distance.

```
> atd <- alongTrackDistance(LA, NY, MS)
> p <- destPoint(LA, b, atd)
> plot(wrld, type='l', xlim=c(-130,-60), ylim=c(22,52))
> lines(gci, col='blue', lwd=2)
> points(rbind(LA, NY), col='red', pch=20, cex=2)
> points(MS[1], MS[2], pch=20, col='blue', cex=2)
> lines(gcIntermediate(LA, p), col='green', lwd=3)
> lines(gcIntermediate(MS, p), col='dark green', lwd=3)
> points(p, pch=20, col='red', cex=2)
> crossTrackDistance(LA, NY, MS)
```

```
      distance
[1,] -549733.7

> distCosine(p, MS)

     distance
[1,] 549733.7
```

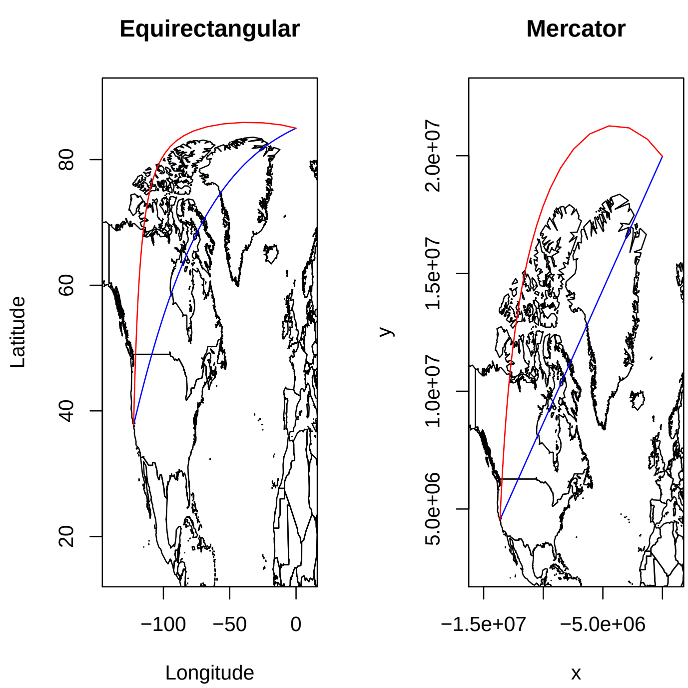


10 Rhumb lines

Rhumb (from the Spanish word for course, 'rumbo') lines are straight lines on a Mercator projection map (and at most latitudes pretty straight on an equirectangular projection (=unprojected lon/lat) map). They were used in navigation because it is easier to follow a constant compass bearing than to continually adjust direction as is needed to follow a great circle, even though rhumb lines are normally longer than great-circle (orthodrome) routes. Most rhumb lines will gradually spiral towards one of the poles.

```
> NP <- c(0, 85)
> bearing(SF, NP)

[1] 5.148274
```

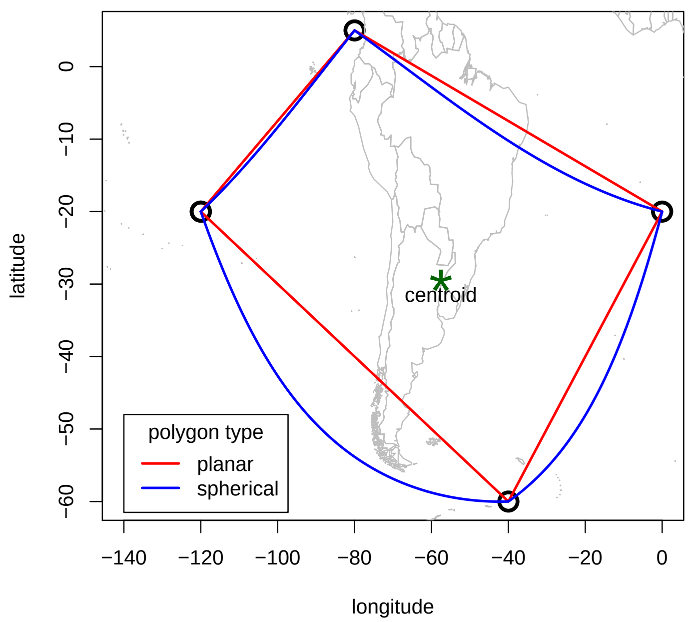
```
> b <- bearingRhumb(SF, NP)
> b
[1] 41.45714
> dc <- distCosine(SF, NP)
> dr <- distRhumb(SF, NP)
> dc / dr
      distance
[1,] 0.8730767
> pr <- destPointRhumb(SF, b, d=round(dr/100) * 1:100)
> pc <- rbind(SF, gcIntermediate(SF, NP), NP)
> par(mfrow=c(1,2))
> data(wrld)
> plot(wrld, type='l', xlim=c(-140,10), ylim=c(15,90), main='Equirectangular')
> lines(pr, col='blue')
> lines(pc, col='red')
> data(merc)
> plot(merc, type='l', xlim=c(-15584729, 1113195),
+                      ylim=c(2500000, 22500000), main='Mercator')
> lines(mercator(pr), col='blue')
> lines(mercator(pc), col='red')
```



11 Characterizing polygons

The package has functions to compute the area, perimeter, centroid, and 'span' of a spherical polygon. One approach to compute these measures is to project the polygons first. Here we directly compute them based on sperhical coordiantes (longitude / latitude), except for centroid, which is computed by projecting the data to the Mercator projection (and inversely projecting the result). The function makePoly inserts additional vertices into a spherical polygon such that it can be plotted (perhaps after first projecting it) more correctly in a plane. Vertices are inserted, where necessary, at a speficief distance. The function is only beneficial for polygons with large inter-vertex distances (in terms of longitude), particularly at high latitudes.

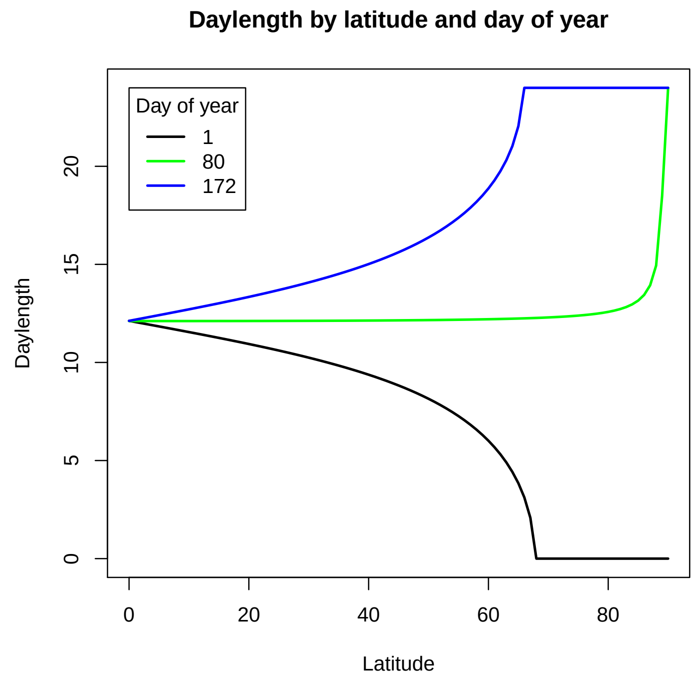
```
> pol <- rbind(c(-120,-20), c(-80,5), c(0, -20), c(-40,-60), c(-120,-20))
> areaPolygon(pol)

[1] 4.920207e+13

> perimeter(pol)

[1] 27357183

> centroid(pol)

           lon       lat
[1,] -57.54653 -28.96994

> span(pol, fun=max)

raster version 1.2-6 (23-June-2010)
         lon     lat
[1,] 9035781 7235767

> nicepoly = makePoly(pol)
> plot(pol, xlab='longitude', ylab='latitude', cex=2, lwd=3, xlim=c(-140, 0))
> lines(wrld, col='grey')
> lines(pol, col='red', lwd=2)
> lines(nicepoly, col='blue', lwd=2)
> points(centroid(pol), pch='*', cex=3, col='dark green')
> text(centroid(pol)-c(0,2.5), 'centroid')
> legend(-140, -48, c('planar','spherical'), lty=1, lwd=2,
+                   col=c('red', 'blue'), title='polygon type')
```



12 Daylength

You can compute daylenght according to the formula by (Forsythe et al, 1995). For any day of the year (an integer between 1 and 365; or a 'Date' object.

```
> as.Date(80, origin='2009-12-31')

[1] "2010-03-21"

> as.Date(172, origin='2009-12-31')

[1] "2010-06-21"

> plot(0:90, daylength(lat=0:90, doy=1), ylim=c(0,24), type='l', xlab='Latitude',
+        ylab='Daylength', main='Daylength by latitude and day of year', lwd=2)
> lines(0:90, daylength(lat=0:90, doy=80), col='green', lwd=2)
> lines(0:90, daylength(lat=0:90, doy=172), col='blue', lwd=2)
> legend(0,24, c('1','80','172'), lty=1, lwd=2, col=c('black', 'green', 'blue'),
+        title='Day of year')
```



13 References

Forsythe, W.C., E.J. Rykiel Jr., R.S. Stahl, H. Wu and R.M. Schoolfield, 1995. A model comparison for daylength as a function of latitude and day of the year. Ecological Modeling 80:87-95.

Sinnott, R.W, 1984. Virtues of the Haversine. Sky and Telescope 68(2): 159

Vincenty, T. 1975. Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations. Survey Review 23(176): 88-93. Available here: `http://www.ngs.noaa.gov/PUBS_LIB/inverse.pdf`