# The pairwise relative semivariogram

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Aug 29, 2011

## 1 Introduction

The general relative variogram (Deutsch and Journel, 1997) is defined as

$$\gamma(h) = \frac{1}{2N_h} \sum_{i=1}^{N_h} \left( \frac{2(Z(s_i) - Z(s_i + h))}{Z(s_i) + Z(s_i + h)} \right)^2 .$$

It is claimed to reveal spatial structure (correlation) better when data are skewed and/or clustered. The `cluster.dat` data set used in this vignette, from the GSLIB distribution[1], seems to confirm this.

From version 1.02 on, R package `gstat` provides computation of the *pairwise relative semivariogram*. The following code provides an example and verification of the computation using direct R code and using the GSLIB program `gamv`.

The following code imports the `cluster.dat` data from GSLIB, which has been converted to have a single-line header containing column names, packaged with the R gstat package, and converts it into a `SpatialPointsDataFrame` object:

```
> library(gstat)
> cluster = read.table(system.file("external/cluster.txt", package = "gstat"),
+     header = TRUE)
> summary(cluster)
```

```
       X               Y             Primary          Secondary
 Min.   : 0.50   Min.   : 0.50   Min.   : 0.060   Min.   : 0.1800
 1st Qu.: 9.50   1st Qu.:14.25   1st Qu.: 0.700   1st Qu.: 0.7875
 Median :25.50   Median :27.00   Median : 2.195   Median : 2.3750
 Mean   :23.32   Mean   :25.61   Mean   : 4.350   Mean   : 4.1402
 3rd Qu.:35.50   3rd Qu.:36.50   3rd Qu.: 5.327   3rd Qu.: 5.5800
 Max.   :48.50   Max.   :48.50   Max.   :58.320   Max.   :22.4600
 Declustering_Weight
 Min.   :0.252
 1st Qu.:0.445
```

[1] F77 source code for Linux, downloaded Aug 28, 2011 from `http://www.gslib.com/`

```
 Median :1.012
 Mean   :1.000
 3rd Qu.:1.416
 Max.   :2.023
```

```
> coordinates(cluster) = ~X + Y
```

The following commands specify a sequence of lag boundaries that correspond to the GSLIB conventions, and compute a regular variogram using these boundaries:

```
> bnd = c(0, 2.5, 7.5, 12.5, 17.5, 22.5, 27.5, 32.5, 37.5, 42.5,
+     47.5, 52.5)
> variogram(Primary ~ 1, cluster, boundaries = bnd)
```

```
     np      dist    gamma dir.hor dir.ver   id
1   149  1.527974 58.07709       0       0 var1
2   624  5.472649 54.09188       0       0 var1
3   989 10.150607 48.85144       0       0 var1
4  1249 15.112173 40.08909       0       0 var1
5  1148 20.033244 42.45081       0       0 var1
6  1367 25.020160 48.60365       0       0 var1
7  1311 29.996102 46.88879       0       0 var1
8  1085 34.907219 44.36890       0       0 var1
9   904 39.876469 47.34666       0       0 var1
10  611 44.716540 38.72725       0       0 var1
11  219 49.387310 30.67908       0       0 var1
```

To compute the relative pairwise variogram, the logical argument `PR` (*pairwise relative*) needs to be set to `TRUE`:

```
> variogram(Primary ~ 1, cluster, boundaries = bnd, PR = TRUE)
```

```
     np      dist     gamma dir.hor dir.ver   id
1   149  1.527974 0.3608431       0       0 var1
2   624  5.472649 0.6307083       0       0 var1
3   989 10.150607 0.8376443       0       0 var1
4  1249 15.112173 0.7769083       0       0 var1
5  1148 20.033244 0.8774599       0       0 var1
6  1367 25.020160 0.8961016       0       0 var1
7  1311 29.996102 0.9002297       0       0 var1
8  1085 34.907219 0.9604305       0       0 var1
9   904 39.876469 0.9055426       0       0 var1
10  611 44.716540 0.7554474       0       0 var1
11  219 49.387310 0.8226759       0       0 var1
```

Figure 1 shows the two variograms, as plots, side by side

## 2 Verification with plain R code

The following R code reproduces the relative pairwise semivariogram values for the first three lags, i.e. 0-2.5, 2.5-7.5 and 7.5-12.5.

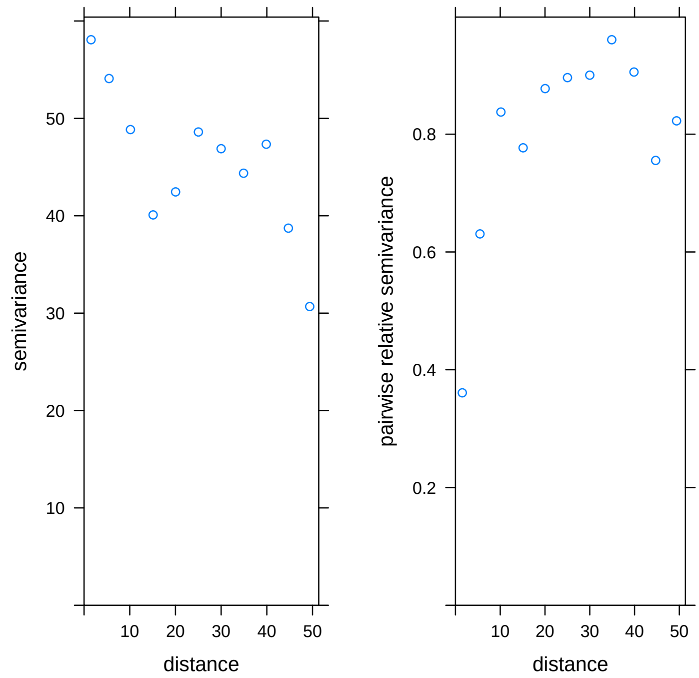


Figure 1: Regular variogram (left) and pairwise relative variogram (right) for the GSLIB data set `cluster.dat`.

```
> z = cluster$Primary
> d = spDists(cluster)
> zd = outer(z, z, "-")
> zs = outer(z, z, "+")
> pr = (2 * zd/zs)^2
> prv = as.vector(pr)
> dv = as.vector(d)
> mean(prv[dv > 0 & dv < 2.5])/2

[1] 0.3608431

> mean(prv[dv > 2.5 & dv < 7.5])/2

[1] 0.6307083

> mean(prv[dv > 7.5 & dv < 12.5])/2

[1] 0.8376443
```

# 3 Verification with GSLIB

In a verification with the GSLIB (Deutsch and Journel, 1997) code of `gamv`, the following file was used:

```
                  Parameters for GAMV
                  *******************

START OF PARAMETERS:
../data/cluster.dat
file with data
1   2   0
columns for X, Y, Z coordinates
1   3
number of varables,column numbers
-1.0e21     1.0e21
trimming limits
gamv.out
file for variogram output
10
number of lags
5.0
lag separation distance
2.5
lag tolerance
1
number of directions
0.0  90.0 50.0   0.0  90.0  50.0
azm,atol,bandh,dip,dtol,bandv
0
standardize sills? (0=no, 1=yes)
```

```
2
number of variograms
1   1   1
tail var., head var., variogram type
1   1   6
tail var., head var., variogram type
```

Running this program with these parameters gave the following output:

```
Semivariogram          tail:Primary      head:Primary       direc-
tion  1
   1         .000       .00000      280        4.35043        4.35043
   2        1.528     58.07709      298        8.62309        8.62309
   3        5.473     54.09188     1248        5.41315        5.41315
   4       10.151     48.85144     1978        4.42758        4.42758
   5       15.112     40.08909     2498        4.25680        4.25680
   6       20.033     42.45081     2296        3.74311        3.74311
   7       25.020     48.60365     2734        4.09575        4.09575
   8       29.996     46.88879     2622        4.15950        4.15950
   9       34.907     44.36890     2170        3.77190        3.77190
  10       39.876     47.34666     1808        4.54173        4.54173
  11       44.717     38.72725     1222        5.15251        5.15251
  12       49.387     30.67908      438        4.56539        4.56539
Pairwise Relative      tail:Primary      head:Primary       direc-
tion  1
   1         .000       .00000      280        4.35043        4.35043
   2        1.528       .36084      298        8.62309        8.62309
   3        5.473       .63071     1248        5.41315        5.41315
   4       10.151       .83764     1978        4.42758        4.42758
   5       15.112       .77691     2498        4.25680        4.25680
   6       20.033       .87746     2296        3.74311        3.74311
   7       25.020       .89610     2734        4.09575        4.09575
   8       29.996       .90023     2622        4.15950        4.15950
   9       34.907       .96043     2170        3.77190        3.77190
  10       39.876       .90554     1808        4.54173        4.54173
  11       44.717       .75545     1222        5.15251        5.15251
  12       49.387       .82268      438        4.56539        4.56539
```

As can be seen, the values in the third column (semivariogram for the first section, pairwise relative semivariogram for the second) correspond to the output generated by `variogram` of package `gstat`. Two differences with respect to the gstat output are:

- for the first lag with distance zero, GSLIB reports that the semivariance value is zero based on 280 point pairs;
- the number of point pairs in GSLIB is double the number reported by gstat.

The ground for these differences seems that the GSLIB `gamv` uses a single routine for computing variograms as well as cross variograms and cross covariances. For cross variograms or covariograms, considering two variables $Z_a$

and $Z_b$ each having $N$ observations, the $N^2$ point pairs $Z_a(s_i), Z_b(s_i + h)$ and $Z_a(s_i + h), Z_b(s_i)$ need to be evaluated, and all contribute information.

For direct (non-cross) variograms or covariograms, $Z_a = Z_b$ and the $N^2$ pairs considered contain the $N$ trivial pairs $(Z(s_i) - Z(s_i))^2 = 0$, which contribute no information, as well as all duplicate pairs, i.e. in addition to $(Z(s_i) - Z(s_i + h))^2$, the identical pair $(Z(s_i + h) - Z(s_i))^2$ is also considered. This leads to correct variogram value estimates, but incorrect unique point pair numbers. (Data set `cluster` contains $N = 140$ observations.)

In contrast, `gstat` considers (and reports) only the number of unique pairs for each lag.

# References


- Deutsch, C.V., A.G. Journel, 1997. GSLIB: Geostatistical Software Library and User's Guide, second edition. Oxford University Press.