Nonparametric mfrd

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This document describes the nonparametric frontier approach in function *mfrd_est*:

mfrd_est(y, x1, x2, c1, c2, t.design = NULL, local = 0.15, front.bw = NA, m = 10, k = 5, kernel = "triangular", ngrid = 250, margin = 0.03, boot = NULL, cluster = NULL, stop.on.error = TRUE)

1 Optimal bandwidth

Assuming that we have an optimal bandwidth b^*, then estimation of treatment effects can proceed similar to the univariate case. We fit a weighted linear model using only points within L_1 distance b^* of the interested frontiers, with weights calculated according to the specified kernel. We denote the linear model by $f(x_1, x_2; b^*)$.

2 Bandwidth evaluation

We want b^* to be optimal in estimating the treatment effects at the frontiers. To evaluate how good a bandwidth b is, we use the mean squared error (MSE) for estimation on a test set:

$$\frac{1}{|S(\delta)|} \sum_{(x_1,x_2)\in S(\delta)} (f(x_1, x_2; b) - y)^2$$

where f is fitted using the training set, and $S(\delta)$ is the test set where all points are within L_1 distance δ of the interest frontiers. This means that:

$$b^* = argmin_b \frac{1}{|S(\delta)|} \sum_{(x_1,x_2)\in S(\delta)} (f(x_1, x_2; b) - y)^2$$

Since we have three treatment effect models (i.e. complete, heterogeneous treatments, treatment only), there is a MSE and hence optimal bandwidth corresponding to each.

Since it is difficult to optimize for b^* exactly, we select the best b from a random sample. In the $mfrd\_est$ function, we draw m choices of b uniformly-at-random from the interval $[0.5, 2.5]$ for the standardized x_1 and x_2, and $m = 10$

as the default value. We set $\delta = 0.25$ to focus on effects at the frontier and also to provide fairer comparison among different b's.

3 Cross-validation for MSE

To calculate the MSE, we implement k-fold cross-validation, with $k = 5$ as the default. In each iteration, the k-th set is used to produce $S(\delta)$, and the remaining $k-1$ sets are used to train the linear model f. The final MSE is the average across all k-folds, and the optimal empirical bandwidth is chosen as the minimizer of this MSE.