trapezoid: The Trapezoidal Distribution

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1 Introduction

The trapezoidal distribution is defined by minimum, lower mode, upper mode, and maximum parameters. The generalized trapezoidal distribution adds three more parameters: the growth rate, decay rate, and boundary ratio parameters. Van Dorp and Kotz[2] and van Dorp and colleagues[1] formally describe the generalized trapezoidal distribution, representing the minimum, lower mode, upper mode, maximum, growth rate, decay rate, and boundary ratio with parameters $a, b, c, d, m, n,$ and α, respectively.

The probability density function of the generalized trapezoidal distribution with parameters $a, b, c, d, m, n,$ and α is given by:

$$f_X(x \mid \theta) = \mathcal{C}(\Theta) \times \begin{cases} \alpha \left(\frac{x-\alpha}{b-\alpha} \right)^{m-1}, & \text{for } a \leq x < b \\ (1-\alpha)\left(\frac{x-b}{c-b} \right) + \alpha, & \text{for } b \leq x < c \\ \left(\frac{d-x}{d-c} \right)^{n-1}, & \text{for } c \leq x \leq d \end{cases}$$

with the normalizing constant $\mathcal{C}(\Theta)$ defined as:

$$\mathcal{C}(\Theta) = \frac{2mn}{2\alpha\,(b-a)\,n + (\alpha+1)\,(c-b)\,mn + 2\,(d-c)\,m}$$

and where the parameter vector $\Theta = \{a, b, c, d, m, n, \alpha\}, a \leq b \leq c \leq d,$ and $m, n, \alpha > 0$.

The trapezoid package provides functions for the probability density function (dtrapezoid), cumulative distribution function (ptrapezoid), quantile function (qtrapezoid), and random generation (rtrapezoid). The parameters $a, b, c, d, m, n,$ and α are specified by the arguments min, mode1, mode2, max, n1, n3, and alpha, respectively. The argument names were chosen to avoid conflicts with names that commonly have specific meaning in R functions, such as c and n.
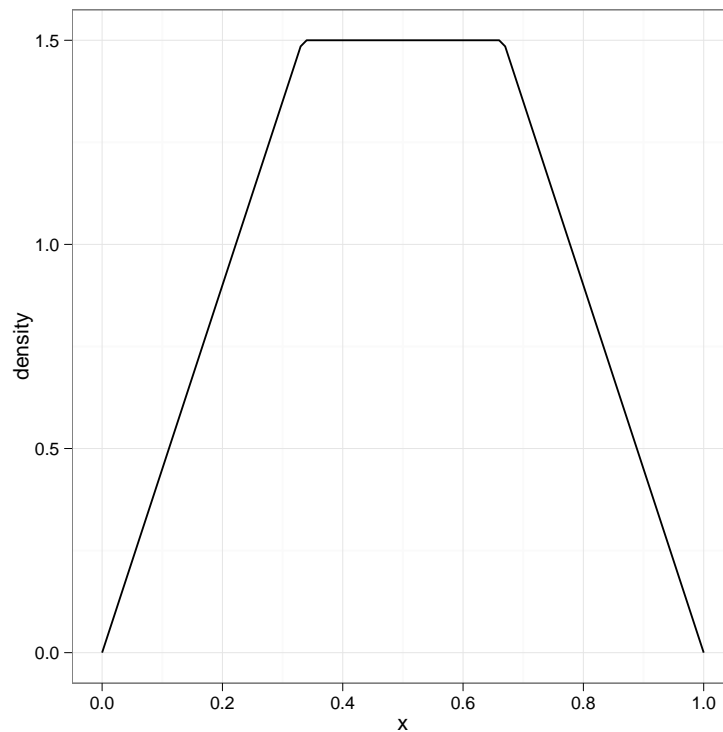
2 Examples

2.1 Trapezoid

The generalized trapezoidal distribution simplifies to the trapezoidal distribution when $m = n = 2$ and $\alpha = 1$.

```
> # plyr and ggplot2 are required for these examples
> require(trapezoid)
> require(plyr)
> require(ggplot2)
> # Trapezoid
> x <- seq(from = 0, to = 1, by = 0.01)
> density <- dtrapezoid(x, min = 0, mode1 = 1/3, mode2 = 2/3, max = 1,
          n1 = 2, n3 = 2, alpha = 1)
> trapezoid <- ggplot(data = data.frame(x, density),
                aes(x = x, y = density)) + geom_line() + theme_bw()
> print(trapezoid)
```

Figure 1: A trapezoidal distribution.
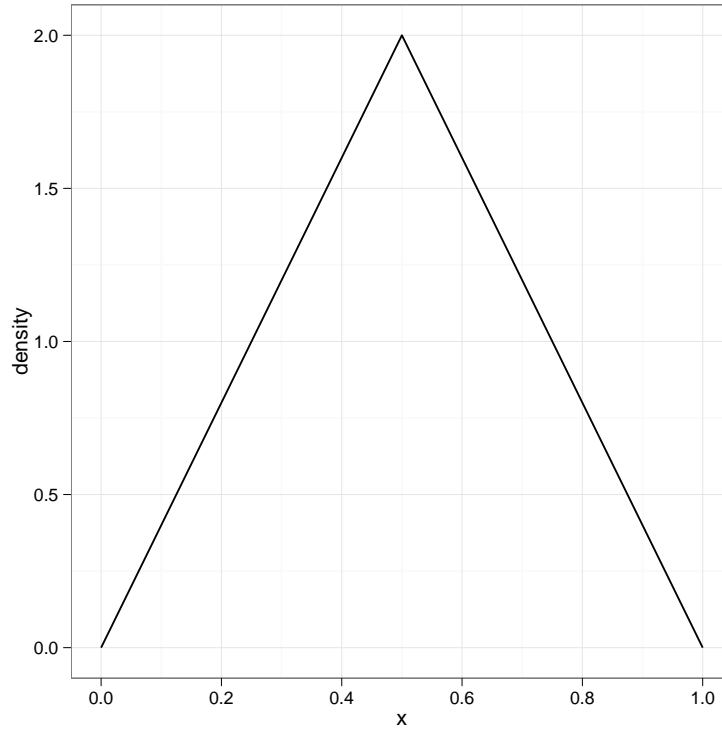

2.2 Triangle

The trapezoidal distribution further simplifies to the triangular distribution when $b = c$.

```
> # Triangle
> x <- seq(from = 0, to = 1, by = 0.01)
> density <- dtrapezoid(x, min = 0, mode1 = 1/2, mode2 = 1/2, max = 1,
        n1 = 2, n3 = 2, alpha = 1)
> triangle <- ggplot(data = data.frame(x, density),
                                aes(x = x, y = density)) + geom_line() + theme_b
> print(triangle)
```

Figure 2: A triangular distribution.

2.3 Generalized trapezoidal distribution

Parameters m, n, and α control the growth rate, decay rate, and boundary ratio, respectively, of the distribution. In the trapezoid package, these parameters are controlled by the n1, n3, and alpha arguments. To demonstrate the effects of these three parameters, van Dorp and Kotz[2] generated eight distributions with varying parameter values. The distributions are approximately re-generated here.

```
> # Generalized trapezoidal distributions
> x <- seq(from = 0, to = 1, by = 0.01)
> # Create a lists of arguments, varying n1, n3, and alpha
> arguments <- list()
> arguments[['A']] <- list(x = x, n1 = 2, n3 = 2, alpha = 0.8)
> arguments[['B']] <- list(x = x, n1 = 1.5, n3 = 1.5, alpha = 1)
> arguments[['C']] <- list(x = x, n1 = 2.5, n3 = 2.5, alpha = 1.5)
```
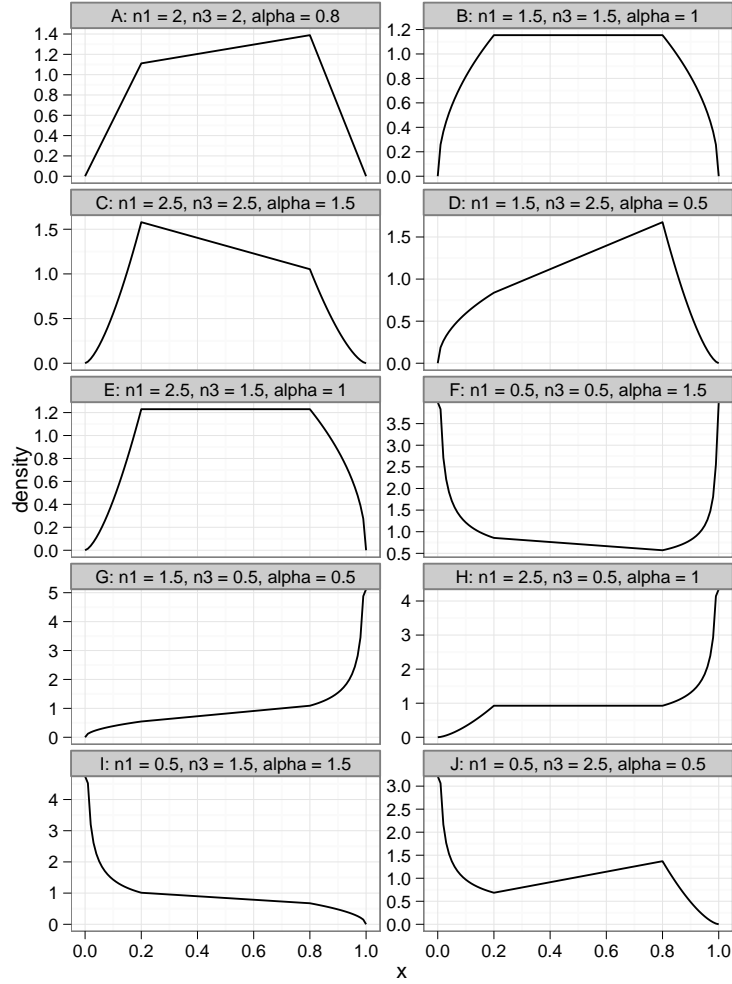
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```
> arguments[['D']] <- list(x = x, n1 = 1.5, n3 = 2.5, alpha = 0.5)
> arguments[['E']] <- list(x = x, n1 = 2.5, n3 = 1.5, alpha = 1)
> arguments[['F']] <- list(x = x, n1 = 0.5, n3 = 0.5, alpha = 1.5)
> arguments[['G']] <- list(x = x, n1 = 1.5, n3 = 0.5, alpha = 0.5)
> arguments[['H']] <- list(x = x, n1 = 2.5, n3 = 0.5, alpha = 1)
> arguments[['I']] <- list(x = x, n1 = 0.5, n3 = 1.5, alpha = 1.5)
> arguments[['J']] <- list(x = x, n1 = 0.5, n3 = 2.5, alpha = 0.5)
> # Calculate the distributions
> plot.data <- ldply(arguments, function(z)
          {
                  x <- z$x
                  density <- dtrapezoid(x = z$x, min = 0, mode1 = 0.2, mode2 = 0.8
                          max = 1, n1 = z$n1, n3 = z$n3, alpha = z$alpha)
                  args <- paste("n1 = ", z$n1, ", n3 = ", z$n3, ", alpha = ", z$al
                          sep="", collapse="")
                  out <- data.frame(x, density, args)
          })
> # Create labels for later use in displaying the arguments on the plots
> plot.data$label <- paste(plot.data$.id, ": ", plot.data$args, sep="")
> # Create plots
> generalizedTrapezoids <- ggplot(data = plot.data, aes(x = x, y = density)) +
                  geom_line() + theme_bw() +
                  facet_wrap(~label, ncol = 2, scales = "free_y")
> print(generalizedTrapezoids)
```

Figure 3: Examples of generalized trapezoidal distributions.

References

[1] J. R. van Dorp, S. C. Rambaud, J. G. Perez, and R. H. Pleguezuelo. An Elicitation Procedure for the Generalized Trapezoidal Distribution with a Uniform Central Stage. *Decision Analysis*, 4(3):156–166, September 2007.

[2] J René van Dorp and Samuel Kotz. Generalized Trapezoidal Distributions. *Metrika*, 58(1):85–97, 2003.