SIMULATION, KRIGING, AND VISUALIZATION

OF CIRCULAR-SPATIAL DATA

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Mathematical Sciences

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Logan, Utah

2009

ABSTRACT

Visualization, Kriging, and Simulation

of Circular-Spatial Data

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Utah State University, 2009

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The circular dataimage is defined by displaying direction as the color at the same direction in a color wheel composed of a sequence of two-color gradients with color continuity between gradients. The resulting image of circular-spatial data is continuous with high resolution. Examples include ocean wind direction, Earth's main magnetic field, and rocket nozzle internal combustion flow. The cosineogram is defined as the mean cosine of the angle between random components of direction as a function of distance between observation locations. It expresses the spatial correlation of circular-spatial data. A circular kriging solution is developed based on a model fitted to the cosineogram. A method for simulating circular random fields is given based on a transformation of a Gaussian random field. It is adaptable to any continuous probability distribution. Circular random fields were implemented for selected circular probability distributions. An R software package was created with functions and documentation.

(391 pages)

ACKNOWLEDGMENTS

Study of the dataimage of Minotte and West (1998), for the imaging of ordered multiple linear variables and observational units, motivated the problem of how to display directional-spatial data as an image, and what this method should be called. I am thankful for the helpful discussions with Dr. Mike Minnotte regarding color wheels. I would also like to thank the JCGS associate editor and referee for their helpful suggestions regarding circular dataimages. I acknowledge and express thanks to ATK Launch Systems, Inc., and NASA for authorizations to display model flow in the Space Shuttle solid rocket motor nozzle in Figure 2-17 (b) and to display nozzle vectoring data in Figure 2-18, and to Technical Artist Alan Eaton for Figure 2-19 (Property number A045477a, ATK Launch Systems, Inc., copyright © 2005) showing the Space Shuttle roll maneuver with left/right solid rocket motor labels added.

Most of the figures were generated in R, versions 2.8.0 (R Development Core Team 2008). R was originally created by Ihaka and Gentleman (1996) and is now a collaborative worldwide effort. The binary distributions of R and R contributor packages are freely downloadable from http://www.r-project.org/, and are supported on Windows (NT, 95 and later) and in some versions for other operating systems. Figure 1-4 was produced using functions of R package Fields (Furrer, Nychka, and Sain 2009), software for simulation of random fields. Figure 1-5, Appendix Figure N-1 (b), and Figure N-2 (a) were produced using functions of R package CircStats (Lund and Agostinelli 2007), software for circular statistics, and manually enhanced. The functions of CircStats were used extensively in the codes written for this dissertation. Figures N-2 (b) and (c) were produced using a demo version of Oriana 2, software for the analysis and display of circular data (Kovach Computing 2004), and manually enhanced. Oriana is available at http://www.kovcomp.co.uk/oriana/oribroc.html. Oriana is supported on Microsoft

Windows 98/Windows NT 4 or later, including Windows ME/2000/XP/2003/Vista. Figures 2-15, 2-16, and N-4 were constructed using functions of R package RGL (Adler 2009), software for 3-D real time visualization. Other R packages used extensively include geoR (Ribeiro and Diggle 2001), and RandomFields (Schlather 2001).

I would like to acknowledge and thank:

ATK Launch Systems, Inc.:

- Dr. Suresh Kulkarni – Provided critical endorsement following rejection of application for the master's program of study.
- Management and supervision – Approved adjusted working hours to accommodate class schedules.

Utah State University professors:

- Dr. Donald Cooley – Head of Computer Science and defense chairman - Suggested programming to have some fun when needing a break.
- Dr. Dan Coster – Provided a listening ear and encouragement.
- Dr. Adele Cutler – Provided a critical evaluation leading to a productive topic of research and a thorough critique of my dissertation.
- Dr. Richard Cutler – Defended me and encouraged high GPA.
- Dr. Robert Heal – Provided a listening ear and encouragement.
- Dr. John Stevens – Suggested important improvements to the dissertation.
- Dr. Jürgen Symanzik, advisor – Provided guidance, persuaded me to remain in the doctoral program of study following withdrawals, and provided critical review.
- Dr. Russell Thompson - Was highly supportive as department head (without him, one withdrawal would have been terminal).

Others:

- Kenneth Johnson, NASA/ NESC Systems Engineering Office – For review of Chapter 1 of the dissertation.
- Terry, wife and mother of 5 children – Sacrificed many hours of companionship and performed many supportive efforts.

William J. Morphet

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LIST OF SYMBOLS, TERMINOLOGY, ACRONYMS

Symbols

σ^2_{CK}	:	Circular kriging variance
Θ	:	Circular random variable (CRV)
θ	:	Observation (realization) or simulation of a CRV
$\boldsymbol{C}$	:	Matrix of cosines of angles between observations of direction
$\boldsymbol{c}$	:	Vector of cosines of angles between observations and unobserved direction to be estimated
$\varsigma(d)$	:	Model of the mean cosine of the angle between random components of direction as a function of distance between observation locations
$\hat{\varsigma}(d)$	:	Cosineogram estimate of $\varsigma(d)$
κ	:	Concentration parameter of the von Mises distribution
n_g	:	Nugget
κ	:	Population concentration about the mean direction for von Mises CRV
μ	:	Population mean resultant vector direction
ρ	:	Population resultant vector mean length and concentration about the mean direction
$\overline{R}_n$	:	Sample resultant mean vector length
$\boldsymbol{x}$	:	Vector of spatial coordinates

New Terminology

- Circular Dataimage
- Cosineocloud
- Cosineogram
- Cosine Model
- Circular Random Field

Terminology from Linear Kriging

- Covariogram
- Covariance Model
- Nugget
- Range
- Sill
- Variogram

LIST OF SYMBOLS, TERMINOLOGY, ACRONYMS

Acronyms

CDF : Cumulative Distribution Function
CRF : Circular Random Field
CRV : Circular Random Variable
CCW: Counterclockwise
GRV : Gaussian Random Variable
GRF : Gaussian Random Field
GUI : Graphical User Interface
GYRB: Green Yellow Red Blue
HSV : Hue Saturation Value
KBWR: BlacK Blue White Red
MAD : Mean Absolute difference
PDF : Probability Density Function
RF : Random Field
RGB : Red Green Blue
RV : Random Variable
YRGB: Yellow Red Green Blue