Basic Reserving Techniques

By Benedict Escoto

This paper is produced mechanically as part of FAViR.
See `http://www.favir.net` for more information.

Contents

1 Introduction

This paper is part of the FAViR series. The first part of the paper presents various basic reserve development methods in R. These methods include:

- Chain Ladder
- Bornhuetter-Ferguson
- Cape-Cod (Standard-Buhlmann)
- Mack Chain Ladder
- Munich Chain Ladder

The last two use code courtesy of Markus Gesmann and estimate reserve uncertainty as well as the expected value. The second part of the paper places these techniques in a popular statistical evaluation [2, 5, 1] framework and presents a couple of basic diagnostics which may indicate which technique is more appropriate for the data in question.

Although the Chain Ladder and Bornhuetter-Ferguson family of reserving methods are well-covered on the actuarial syllabus [3], this R implementation may be useful for several reasons. First, if R is used for other methods, it may be convenient to use basic methods in R as a check. Second, this paper may facilitate the production of automated reserving reports. Third, basic reserving diagnostics and uncertainty measurements can be time consuming to program and display.

2 Original Data

This chapter does not contain any techniques, but simply prints the input data used for later methods. The reserving techniques in this paper require only basic information:

1. Paid and case-incurred losses by development age and origin
2. Earned premium by origin
3. A priori loss by origin (for the Bornhuetter-Ferguson method)

where "origin" can be accident year, policy year, etc.

All the required data is shown in this section. Figure 1 is the input triangle showing incurred losses by accident year and development month. Figure 2 is the corresponding record of paid losses. Figure 3 shows the premium and a priori loss estimates by accident year.

Accident Year	Incurred Loss by Development Age											
	3	15	27	39	51	63	75	87	99	111	123	135
1995	44	1,331	3,319	4,020	4,232	4,252	4,334	4,369	4,386	4,395	4,401	4,399
1996	42	1,244	3,508	4,603	4,842	4,970	5,059	5,083	5,155	5,205	5,205	
1997	17	1,088	3,438	4,169	4,371	4,482	4,626	4,734	4,794	4,804		
1998	10	781	3,135	4,085	4,442	4,777	4,914	5,110	5,176			
1999	13	937	3,506	4,828	5,447	5,790	6,112	6,295				
2000	2	751	2,639	3,622	3,931	4,077	4,244					
2001	4	1,286	3,570	4,915	5,377	5,546						
2002	2	911	5,023	6,617	7,194							
2003	3	1,398	4,021	4,825								
2004	4	1,130	3,981									
2005	21	915										
2006	13											

Figure 1: Incurred Loss Triangle

Accident Year	Paid Loss by Development Age											
	3	15	27	39	51	63	75	87	99	111	123	135
1995	3	503	2,474	3,719	4,094	4,194	4,303	4,350	4,382	4,394	4,394	4,398
1996	1	465	2,621	4,122	4,618	4,882	4,997	5,041	5,111	5,172	5,191	
1997	1	534	2,541	3,807	4,192	4,374	4,544	4,679	4,761	4,787		
1998	1	329	2,204	3,673	4,242	4,616	4,827	5,051	5,145			
1999	1	399	2,496	4,304	5,197	5,674	6,031	6,244				
2000	1	328	1,849	3,124	3,693	3,966	4,164					
2001	1	443	2,566	4,208	5,074	5,474						
2002	1	401	3,078	5,459	6,748							
2003	1	326	2,372	4,132								
2004	4	524	2,784									
2005	1	323										
2006	1											

Figure 2: Paid Loss Triangle

Accident Year	Earned Premium	A Priori Loss	A Priori Loss Ratio
1995	6,000	4,800	80.0
1996	6,000	4,800	80.0
1997	6,000	4,800	80.0
1998	6,000	4,800	80.0
1999	6,000	4,800	80.0
2000	6,000	4,800	80.0
2001	6,000	4,800	80.0
2002	6,000	4,800	80.0
2003	6,000	4,800	80.0
2004	6,000	4,800	80.0
2005	6,000	4,800	80.0
2006	6,000	4,800	80.0
Avg	6,000	4,800	80.0

Figure 3: Premium and A Priori Loss

3 Basic Methods

This chapter includes the traditional Chain Ladder and Bornhuetter-Ferguson methods. They are performed separately on paid and case-incurred losses.

3.1 LDF Selection

Figure 4 shows LDFs derived from paid loss triangles in the traditional manner. Below we will use the weighted average LDFs as our selected paid age-to-age factors. LDFs for incurred loss are presented in figure 5.

3.2 Tail Selection

One family of methods estimates tail factors by fitting the age-to-age factors for older years to various curves. The tail factor can be found by extrapolating the curve to infinity. This section performs this fitting separately for paid and incurred loss.

For paid loss, the factors in 4 are used. The trailing LDFs used for fitting are shown in figure 6 and the results are shown in figure 7.

For incurred loss, the factors are taken from 5. The trailing LDFs used for fitting are shown in figure 8 and the results are shown in figure 9.

3.3 Final LDF Selection

Selecting the modified McClenahan tail factor, we arrive at the final LDFs to ultimate. Paid LDFs are in figure 10; figure 11 has incurred LDFs to ultimate.

3.4 Chain Ladder

Figure 12 shows the results by accident year of apply the basic chain ladder technique on paid losses. Figure 13 shows the results by accident year of apply the basic chain ladder technique on incurred losses.

3.5 Bornhuetter-Ferguson

Basic reserves by accident year according to the Bornhuetter-Ferguson method applied to paid loss are shown in figure 14. Figure 15 is the corresponding incurred loss exhibit.

3.6 Cape Cod (Stanard-Buhlmann)

The Cape Cod technique has two stages. The first, picking a prior loss ratio, is shown in figure 16 for paid loss and in figure 18 for incurred loss. The resulting loss ratio, as shown in the last row, is the ratio of the sum of latest diagonals with the used-up premium.

Accident Year	Paid Loss by Development Age											
	3	15	27	39	51	63	75	87	99	111	123	135
1995	3	503	2,474	3,719	4,094	4,194	4,303	4,350	4,382	4,394	4,394	4,398
1996	1	465	2,621	4,122	4,618	4,882	4,997	5,041	5,111	5,172	5,191	
1997	1	534	2,541	3,807	4,192	4,374	4,544	4,679	4,761	4,787		
1998	1	329	2,204	3,673	4,242	4,616	4,827	5,051	5,145			
1999	1	399	2,496	4,304	5,197	5,674	6,031	6,244				
2000	1	328	1,849	3,124	3,693	3,966	4,164					
2001	1	443	2,566	4,208	5,074	5,474						
2002	1	401	3,078	5,459	6,748							
2003	1	326	2,372	4,132								
2004	4	524	2,784									
2005	1	323										
2006	1											

Accident Year	Age to Age Loss Development Factors										
	3 to 15	15 to 27	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135
1995	167.67	4.92	1.50	1.10	1.02	1.03	1.01	1.01	1.00	1.00	1.00
1996	465.00	5.64	1.57	1.12	1.06	1.02	1.01	1.01	1.01	1.00	
1997	534.00	4.76	1.50	1.10	1.04	1.04	1.03	1.02	1.01		
1998	329.00	6.70	1.67	1.15	1.09	1.05	1.05	1.02			
1999	399.00	6.26	1.72	1.21	1.09	1.06	1.04				
2000	328.00	5.64	1.69	1.18	1.07	1.05					
2001	443.00	5.79	1.64	1.21	1.08						
2002	401.00	7.68	1.77	1.24							
2003	326.00	7.28	1.74								
2004	131.00	5.31									
2005	323.00										

	Averaged Age-to-Age LDFs										
	3 to 15	15 to 27	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135
Average	349.70	6.00	1.65	1.16	1.07	1.04	1.03	1.01	1.01	1.00	1.00
Avg xHi,Lo	353.52	5.94	1.65	1.16	1.07	1.04	1.03	1.02	1.01	1.00	1.00
Avg Last 5	324.80	6.34	1.71	1.20	1.08	1.04	1.03	1.01	1.01	1.00	1.00
Weighted Avg	285.94	5.88	1.65	1.17	1.07	1.04	1.03	1.01	1.01	1.00	1.00
Weighted Last 5	252.12	6.26	1.72	1.20	1.08	1.04	1.03	1.01	1.01	1.00	1.00

Figure 4: Traditional LDF Exhibit based on Paid Loss

Accident Year	Incurred Loss by Development Age											
	3	15	27	39	51	63	75	87	99	111	123	135
1995	44	1,331	3,319	4,020	4,232	4,252	4,334	4,369	4,386	4,395	4,401	4,399
1996	42	1,244	3,508	4,603	4,842	4,970	5,059	5,083	5,155	5,205	5,205	
1997	17	1,088	3,438	4,169	4,371	4,482	4,626	4,734	4,794	4,804		
1998	10	781	3,135	4,085	4,442	4,777	4,914	5,110	5,176			
1999	13	937	3,506	4,828	5,447	5,790	6,112	6,295				
2000	2	751	2,639	3,622	3,931	4,077	4,244					
2001	4	1,286	3,570	4,915	5,377	5,546						
2002	2	911	5,023	6,617	7,194							
2003	3	1,398	4,021	4,825								
2004	4	1,130	3,981									
2005	21	915										
2006	13											

Accident Year	Age to Age Loss Development Factors										
	3 to 15	15 to 27	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135
1995	30.25	2.49	1.21	1.05	1.00	1.02	1.01	1.00	1.00	1.00	1.00
1996	29.62	2.82	1.31	1.05	1.03	1.02	1.00	1.01	1.01	1.00	
1997	64.00	3.16	1.21	1.05	1.03	1.03	1.02	1.01	1.00		
1998	78.10	4.01	1.30	1.09	1.08	1.03	1.04	1.01			
1999	72.08	3.74	1.38	1.13	1.06	1.06	1.03				
2000	375.50	3.51	1.37	1.09	1.04	1.04					
2001	321.50	2.78	1.38	1.09	1.03						
2002	455.50	5.51	1.32	1.09							
2003	466.00	2.88	1.20								
2004	282.50	3.52									
2005	43.57										

	Averaged Age-to-Age LDFs										
	3 to 15	15 to 27	27 to 39	39 to 51	51 to 63	63 to 75	75 to 87	87 to 99	99 to 111	111 to 123	123 to 135
Average	201.69	3.44	1.30	1.08	1.04	1.03	1.02	1.01	1.00	1.00	1.00
Avg xHi,Lo	191.44	3.30	1.30	1.08	1.04	1.03	1.02	1.01	1.00	1.00	1.00
Avg Last 5	313.81	3.64	1.33	1.10	1.05	1.04	1.02	1.01	1.00	1.00	1.00
Weighted Avg	72.67	3.33	1.30	1.08	1.04	1.03	1.02	1.01	1.00	1.00	1.00
Weighted Last 5	165.88	3.51	1.32	1.10	1.05	1.04	1.02	1.01	1.00	1.00	1.00

Figure 5: Traditional LDF Exhibit Based on Incurred Loss

87 to 99	99 to 111	111 to 123	123 to 135
1	1	1	1

Figure 6: Tail Factors to Fit: Paid Loss

Method	Tail Factor to Ultimate
McClenahan Method (exponential)	1
Modified McClenahan Method	1
Exponential Decay of LDFs to 1.0	1
Sherman Method (inverse power law)	1

Figure 7: Results of Tail Fitting: Paid Loss

87 to 99	99 to 111	111 to 123	123 to 135
1	1	1	1

Figure 8: Tail Factors to Fit: Incurred Loss

Method	Tail Factor to Ultimate
McClenahan Method (exponential)	1
Modified McClenahan Method	1
Exponential Decay of LDFs to 1.0	1
Sherman Method (inverse power law)	1

Figure 9: Results of Tail Fitting: Incurred Loss

	Development Age											
	3	15	27	39	51	63	75	87	99	111	123	135
LDFs to Ultimate	3785.67	13.24	2.25	1.37	1.17	1.10	1.05	1.03	1.01	1.01	1.00	1.00

Figure 10: Selected LDFs to Ultimate: Paid Loss

	Development Age											
	3	15	27	39	51	63	75	87	99	111	123	135
LDFs to Ultimate	3785.67	13.24	2.25	1.37	1.17	1.10	1.05	1.03	1.01	1.01	1.00	1.00

Figure 11: Selected LDFs to Ultimate: Incurred Loss

Accident Year	Development Age	Latest Diagonal	LDF to Ultimate	Percent Developed	Ultimate Loss
1995	135	4,398	1.00	99.8	4,409
1996	123	5,191	1.00	99.7	5,209
1997	111	4,787	1.01	99.5	4,813
1998	99	5,145	1.01	98.8	5,209
1999	87	6,244	1.03	97.4	6,413
2000	75	4,164	1.05	94.8	4,392
2001	63	5,474	1.10	91.0	6,015
2002	51	6,748	1.17	85.3	7,908
2003	39	4,132	1.37	73.1	5,655
2004	27	2,784	2.25	44.4	6,273
2005	15	323	13.24	7.6	4,276
2006	3	1	3785.67	0.0	3,786

Figure 12: Results of Chain Ladder Method on Paid Loss

Accident Year	Development Age	Latest Diagonal	LDF to Ultimate	Percent Developed	Ultimate Loss
1995	135	4,399	1.00	99.9	4,403
1996	123	5,205	1.00	99.9	5,208
1997	111	4,804	1.00	99.9	4,809
1998	99	5,176	1.01	99.4	5,207
1999	87	6,295	1.02	98.3	6,403
2000	75	4,244	1.04	96.2	4,411
2001	63	5,546	1.07	93.1	5,956
2002	51	7,194	1.12	89.7	8,021
2003	39	4,825	1.21	83.0	5,815
2004	27	3,981	1.56	64.0	6,218
2005	15	915	5.20	19.2	4,758
2006	3	13	377.83	0.3	4,912

Figure 13: Results of Chain Ladder on Incurred Loss

Accident Year	Development Age	Latest Diagonal	LDF to Ultimate	Percent Developed	A Priori Loss	BF Ultimate Loss
1995	135	4,398	1	99.8	4,800	4,410
1996	123	5,191	1	99.7	4,800	5,207
1997	111	4,787	1	99.5	4,800	4,813
1998	99	5,145	1	98.8	4,800	5,204
1999	87	6,244	1	97.4	4,800	6,371
2000	75	4,164	1	94.8	4,800	4,413
2001	63	5,474	1	91.0	4,800	5,906
2002	51	6,748	1	85.3	4,800	7,452
2003	39	4,132	1	73.1	4,800	5,425
2004	27	2,784	2	44.4	4,800	5,454
2005	15	323	13	7.6	4,800	4,760
2006	3	1	3,786	0.0	4,800	4,800

Figure 14: Results of Bornhuetter-Ferguson Method on Paid Loss

Accident Year	Development Age	Latest Diagonal	LDF to Ultimate	Percent Developed	A Priori Loss	BF Ultimate Loss
1995	135	4,399	1	99.9	4,800	4,404
1996	123	5,205	1	99.9	4,800	5,207
1997	111	4,804	1	99.9	4,800	4,809
1998	99	5,176	1	99.4	4,800	5,204
1999	87	6,295	1	98.3	4,800	6,376
2000	75	4,244	1	96.2	4,800	4,426
2001	63	5,546	1	93.1	4,800	5,876
2002	51	7,194	1	89.7	4,800	7,689
2003	39	4,825	1	83.0	4,800	5,642
2004	27	3,981	2	64.0	4,800	5,708
2005	15	915	5	19.2	4,800	4,792
2006	3	13	378	0.3	4,800	4,800

Figure 15: Results of Bornhuetter-Ferguson Method on Incurred Loss

This loss ratio is then used as the a priori loss ratio in the Bornhuetter-Ferguson technique to determine the ultimate loss. Figure 17 demonstrates this for paid loss. Incurred loss is shown in figure 19.

This loss ratio is applied in figure 17 on paid loss to obtain the ultimate loss according to the Cape Cod method.

Accident Year	Latest Diagonal	LDF to Ultimate	Total Premium	Used-Up Premium	Expected Loss Ratio
1995	4,398	1.00	6,000	5,985	73.5
1996	5,191	1.00	6,000	5,980	86.8
1997	4,787	1.01	6,000	5,968	80.2
1998	5,145	1.01	6,000	5,927	86.8
1999	6,244	1.03	6,000	5,842	106.9
2000	4,164	1.05	6,000	5,689	73.2
2001	5,474	1.10	6,000	5,460	100.2
2002	6,748	1.17	6,000	5,120	131.8
2003	4,132	1.37	6,000	4,384	94.3
2004	2,784	2.25	6,000	2,663	104.5
2005	323	13.24	6,000	453	71.3
2006	1	3785.67	6,000	2	63.1
Total	49,391		72,000	53,472	92.4

Figure 16: Cape Cod Loss Ratio Selection: Paid Loss

Accident Year	Development Age	Latest Diagonal	LDF to Ultimate	Percent Developed	A Priori Loss	Cape Cod Ultimate
1995	135	4,398	1	99.8	5,542	4,412
1996	123	5,191	1	99.7	5,542	5,210
1997	111	4,787	1	99.5	5,542	4,817
1998	99	5,145	1	98.8	5,542	5,213
1999	87	6,244	1	97.4	5,542	6,390
2000	75	4,164	1	94.8	5,542	4,451
2001	63	5,474	1	91.0	5,542	5,972
2002	51	6,748	1	85.3	5,542	7,561
2003	39	4,132	1	73.1	5,542	5,625
2004	27	2,784	2	44.4	5,542	5,866
2005	15	323	13	7.6	5,542	5,446
2006	3	1	3,786	0.0	5,542	5,542

Figure 17: Results of Cape Cod Method on Paid Loss

Accident Year	Latest Diagonal	LDF to Ultimate	Total Premium	Used-Up Premium	Expected Loss Ratio
1995	4,399	1.00	6,000	5,994	73.4
1996	5,205	1.00	6,000	5,997	86.8
1997	4,804	1.00	6,000	5,993	80.2
1998	5,176	1.01	6,000	5,964	86.8
1999	6,295	1.02	6,000	5,899	106.7
2000	4,244	1.04	6,000	5,773	73.5
2001	5,546	1.07	6,000	5,587	99.3
2002	7,194	1.12	6,000	5,381	133.7
2003	4,825	1.21	6,000	4,979	96.9
2004	3,981	1.56	6,000	3,841	103.6
2005	915	5.20	6,000	1,154	79.3
2006	13	377.83	6,000	16	81.9
Total	52,597		72,000	56,579	93.0

Figure 18: Cape Cod Loss Ratio Selection: Incurred Loss

Accident Year	Development Age	Latest Diagonal	LDF to Ultimate	Percent Developed	A Priori Loss	Cape Cod Ultimate
1995	135	4,399	1	99.9	5,578	4,404
1996	123	5,205	1	99.9	5,578	5,208
1997	111	4,804	1	99.9	5,578	4,810
1998	99	5,176	1	99.4	5,578	5,209
1999	87	6,295	1	98.3	5,578	6,389
2000	75	4,244	1	96.2	5,578	4,455
2001	63	5,546	1	93.1	5,578	5,930
2002	51	7,194	1	89.7	5,578	7,769
2003	39	4,825	1	83.0	5,578	5,774
2004	27	3,981	2	64.0	5,578	5,988
2005	15	915	5	19.2	5,578	5,420
2006	3	13	378	0.3	5,578	5,576

Figure 19: Results of Cape Cod Method on Incurred Loss

4 The ChainLadder Package

This chapter uses the ChainLadder R package by Markus Gesmann. See `http://code.google.com/p/chainladder/` for more information on this package.

4.1 Mack Chain Ladder

Thomas Mack derived in 1993 a very straightforward stochastic model under which the traditional Chain Ladder method would be reasonable.[4] Mack's model can be used to calculate the standard deviation of bulk reserves.

4.1.1 Paid Loss

The results of Mack's Chain Ladder fitted model applied to paid loss are summarized in figure 20. For each origin period, the expected ultimate should exactly match the simple chain ladder results in figure 12. The expected development is graphed in figure 21. Figure 22 shows standardized residuals with a smoothing guide line. Because chain ladder methods choose different factors for each development age, the development age factors should be unbiased. However, if the other plots show any significant trends, it may indicate that the assumptions behind the chain ladder method do not hold. Barnett and Zehnwirth in [1] discuss the interpretation of residual plots.

4.1.2 Incurred Loss

The results of Mack's Chain Ladder fitted model applied to case-incurred loss are summarized in figure 23. For each origin period, the expected ultimate should exactly match the simple chain ladder results in figure 13. As with the paid residual plot, bias or trends in figure 25 may indicate a failure of model assumptions.

4.2 Munich Chain Ladder

The Munich Chain Ladder technique is also included in the ChainLadder package by Markus Gesmann. Typically running chain ladder techniques separately on paid and incurred triangles results in different ultimate loss picks. The Munich Chain Ladder incorporates information from both triangles when selecting LDFs. The results of the method are shown in figure 26.

The central idea of the Munich Chain Ladder is that the paid/incurred loss ratios at the beginning of each development period provide extra information about the loss development in that period. For instance, if the paid/incurred ratio is unusually low, greater than normal paid development is more likely. Figure 27 shows how paid and incurred residuals depend on

Accident Year	Latest Diagonal	Percent Developed	Mack Ultimate	Bulk Reserve	Mack Standard Error	CV of Bulk Reserves
1995	4,398	99.8	4,409	11	14	1
1996	5,191	99.7	5,209	18	18	1
1997	4,787	99.5	4,813	26	23	1
1998	5,145	98.8	5,209	64	37	1
1999	6,244	97.4	6,413	169	53	0
2000	4,164	94.8	4,392	228	90	0
2001	5,474	91.0	6,015	541	141	0
2002	6,748	85.3	7,908	1,160	234	0
2003	4,132	73.1	5,655	1,523	335	0
2004	2,784	44.4	6,273	3,489	531	0
2005	323	7.6	4,276	3,953	939	0
2006	1	0.0	3,786	3,785	2,481	1
Total	49,391		64,356	14,965	2,817	0

Figure 20: Mack Chain Ladder Results: Paid Loss

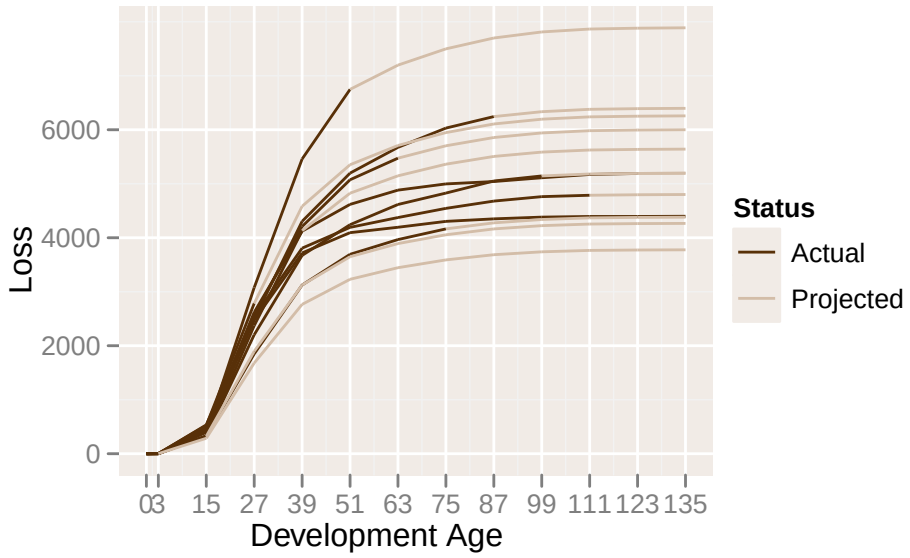

Figure 21: Mack Actual and Predicted Development on Paid Loss

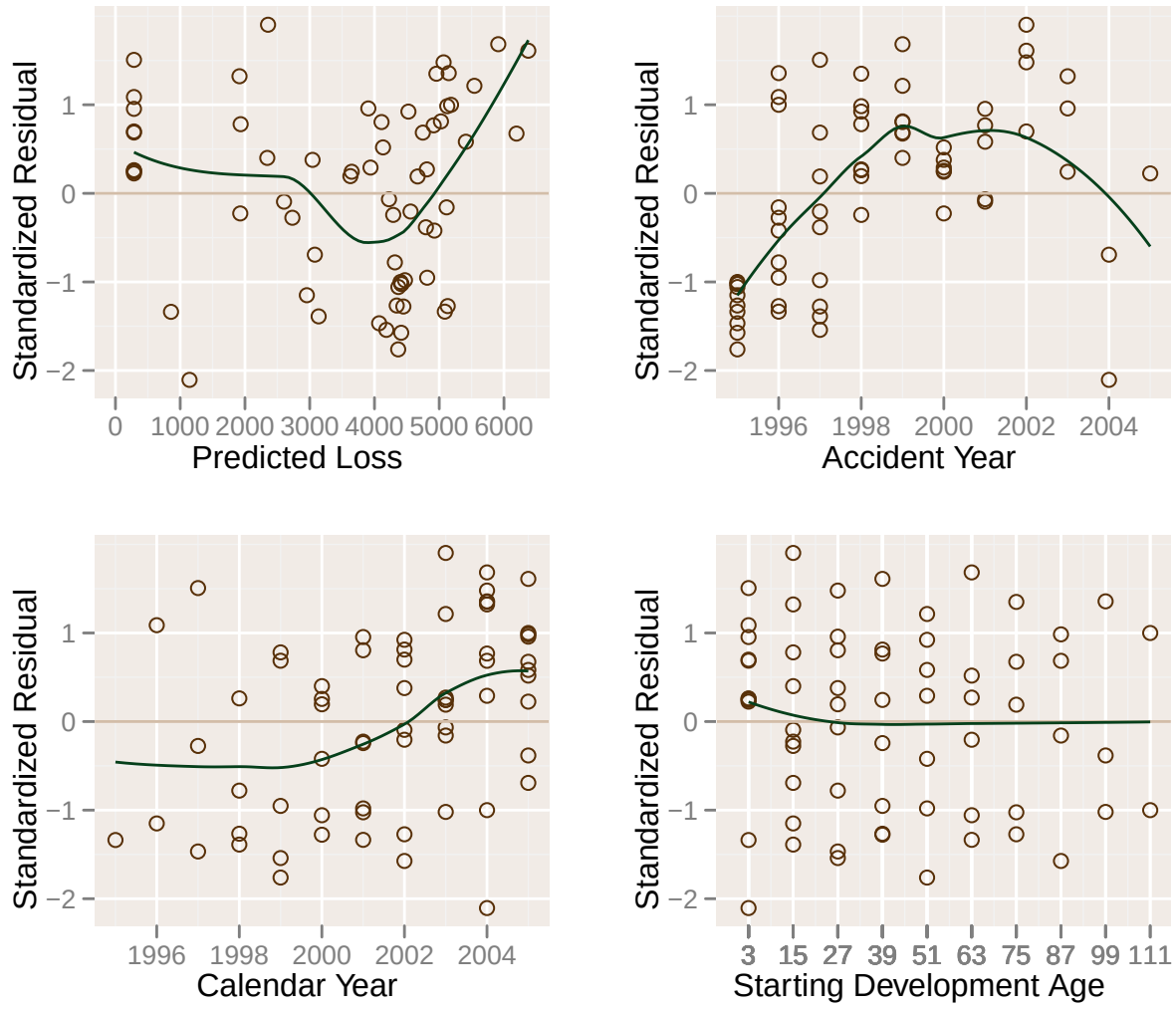

Figure 22: Mack Model Residuals: Paid Loss

Accident Year	Latest Diagonal	Percent Developed	Mack Ultimate	Bulk Reserve	Mack Standard Error	CV of Bulk Reserves
1995	4,399	99.9	4,403	4	5	1
1996	5,205	99.9	5,208	3	6	2
1997	4,804	99.9	4,809	5	8	1
1998	5,176	99.4	5,207	31	27	1
1999	6,295	98.3	6,403	108	43	0
2000	4,244	96.2	4,411	167	82	0
2001	5,546	93.1	5,956	410	131	0
2002	7,194	89.7	8,021	827	226	0
2003	4,825	83.0	5,815	990	241	0
2004	3,981	64.0	6,218	2,237	429	0
2005	915	19.2	4,758	3,843	1,412	0
2006	13	0.3	4,912	4,899	7,782	2
Total	52,597		66,120	13,523	7,964	1

Figure 23: Mack Chain Ladder Results: Incurred Loss

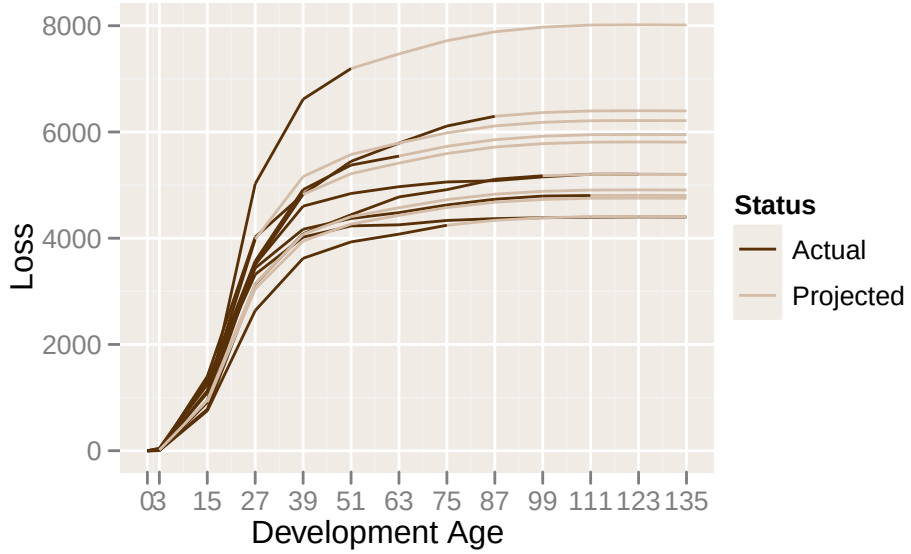

Figure 24: Mack Actual and Predicted Development on Incurred Loss

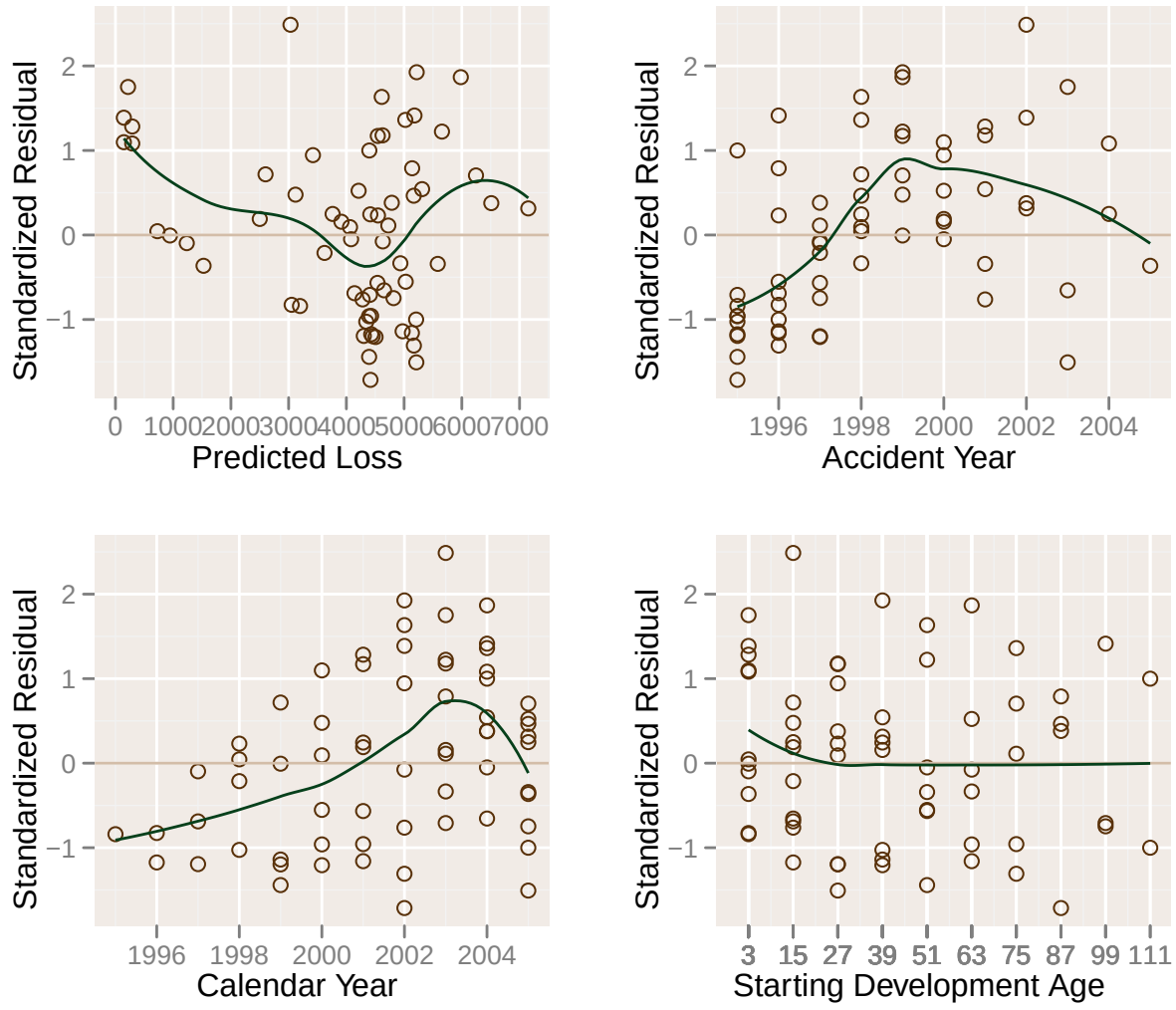

Figure 25: Mack Model Residuals: Incurred Loss

the previous ratios of paid to incurred loss. Munich method adjusts the expected paid development based on the slope of the line in the left graph. The expected incurred development is adjusted by the right line's slope.

Accident Year	Latest Paid	Latest Incurred	Latest P/I (%)	Ultimate Paid	Ultimate Incurred	Ultimate P/I (%)
1995	4,398	4,399	100.0	4,409	4,403	100.1
1996	5,191	5,205	99.7	5,222	5,207	100.3
1997	4,787	4,804	99.6	4,821	4,809	100.3
1998	5,145	5,176	99.4	5,220	5,207	100.3
1999	6,244	6,295	99.2	6,419	6,403	100.3
2000	4,164	4,244	98.1	4,421	4,410	100.3
2001	5,474	5,546	98.7	5,974	5,959	100.3
2002	6,748	7,194	93.8	8,034	8,014	100.3
2003	4,132	4,825	85.6	5,822	5,807	100.3
2004	2,784	3,981	69.9	6,236	6,220	100.3
2005	323	915	35.3	4,733	4,721	100.3
2006	1	13	7.7	4,828	4,816	100.3
Totals	49,391	52,597	93.9	66,139	65,976	100.2

Figure 26: Munich Chain Ladder Results

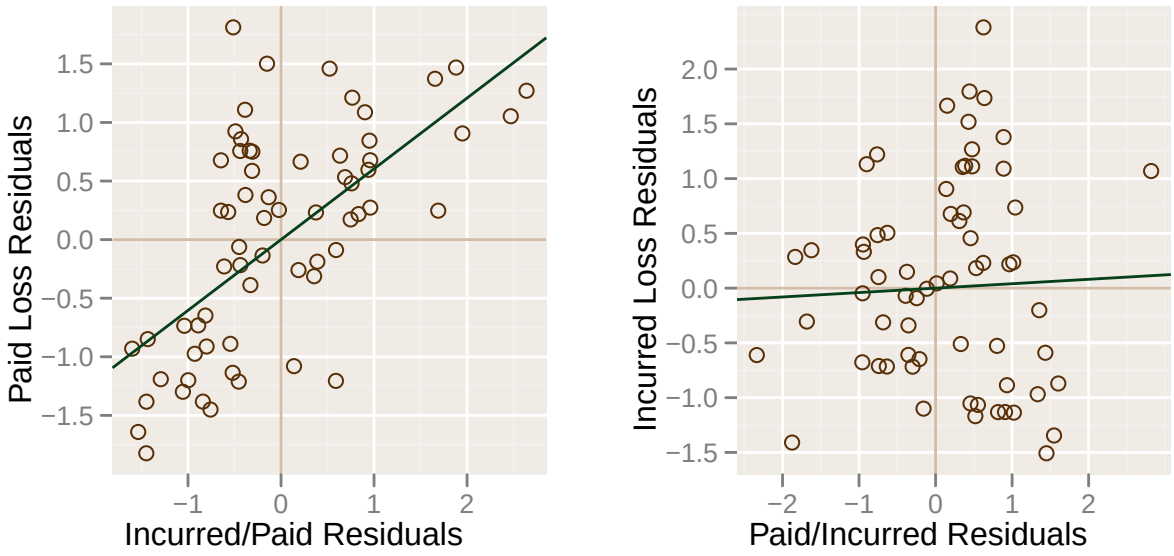

Figure 27: Munich Chain Ladder Standardized Residuals

5 Assumption Testing

The choice of a development method and age-to-age factors can be considered a special case of linear regression. Each development period is a separate regression where loss development, the dependent variable, depends on the starting loss, the independent variable. Once reserving is construed as linear regression, we can use the standard plots and measures of regression to test the assumptions of our methods.

Figure 28 illustrates the results of running three linear regressions on each age period's paid loss. Each regression corresponds to a different reserving model. If the Bornhuetter-Ferguson or Cape-Cod model is correct, the expected development during each period is independent of the previous development. Thus the regression line should be horizontal. According to the Chain Ladder method, the development should be proportional to the current total loss; thus the regression line is sloped but should have no intercept term. Finally we can consider the possibility that the expected development has both a slope and intercept term.

Figure 29 shows common regression statistics on paid loss by development period. The R^2 of the intercept-only model will always be 0% by definition. A positive R^2 for the link-only (chain ladder) model means that it "explains" more of the variation than the constant development model does. If we include both an intercept and a link parameter, the t- and p-values of each may indicate which fits the data better. The further the t-value is away from 0 and the smaller the p-value, the more important that parameter is to loss development.

Figures 30 and 31 are the analogous exhibits covering regression on incurred loss.

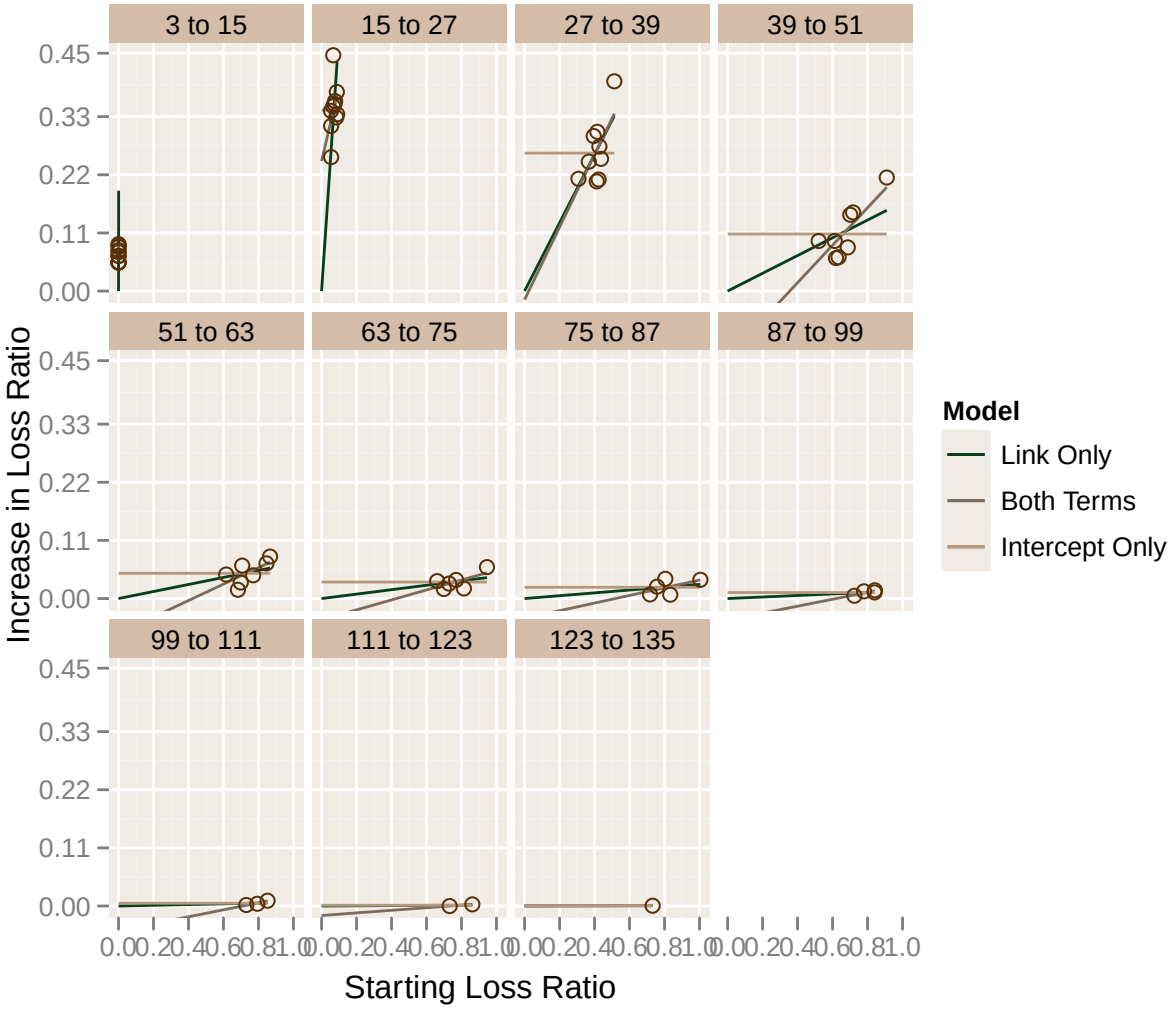

Figure 28: Regression by Development Period: Paid Loss

Development Period	Link Only R^2 %	Fit Results: Link and Intercept Model				
		R^2 %	Link t-Value	Intercept t-Value	Link Power p-val %	Intercept p-val %
3 to 15	-448.1	13.7	1	7	26.2	0.0
15 to 27	-72.2	13.6	1	3	29.5	2.3
27 to 39	45.1	45.3	2	-0	4.7	89.3
39 to 51	44.3	61.8	3	-2	2.1	14.8
51 to 63	28.4	42.5	2	-1	11.2	31.8
63 to 75	28.8	41.9	2	-1	16.5	39.6
75 to 87	16.9	28.2	1	-1	35.8	54.1
87 to 99	25.8	65.9	2	-2	18.8	26.5
99 to 111	18.3	93.0	4	-3	17.0	18.9
111 to 123	15.0	100.0				
123 to 135		0.0				

Figure 29: Regression Statistics: Paid Loss

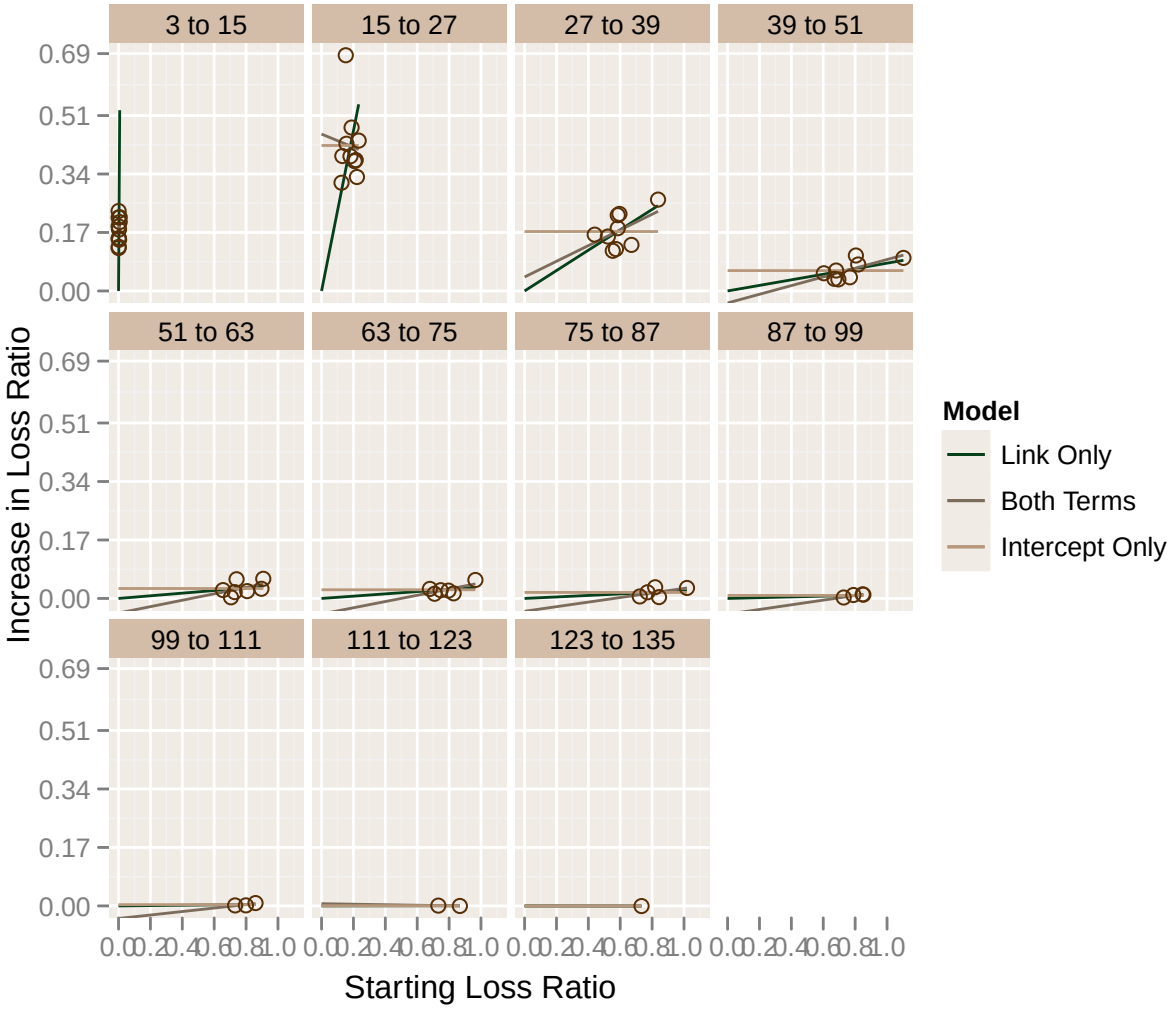

Figure 30: Regression by Development Period: Incurred Loss

Development Period	Link Only R^2 %	Fit Results: Link and Intercept Model				
		R^2 %	Link t-Value	Intercept t-Value	Link Power p-val %	Intercept p-val %
3 to 15	-1184.5	1.2	0	10	75.3	0.0
15 to 27	-82.2	0.5	-0	3	85.3	3.3
27 to 39	21.4	23.5	1	0	18.6	67.0
39 to 51	38.6	44.5	2	-1	7.1	45.3
51 to 63	12.9	19.9	1	-1	31.6	53.9
63 to 75	25.7	43.4	2	-1	15.5	32.6
75 to 87	15.6	28.8	1	-1	35.1	51.1
87 to 99	26.2	87.8	4	-3	6.3	8.7
99 to 111	12.9	72.9	2	-1	34.8	37.6
111 to 123	-18.4	100.0				
123 to 135		0.0				

Figure 31: Regression Statistics: Incurred Loss

6 Summary of Results

This section simply compiles the results of the various methods covered earlier. Figures 34 and following show the results in tabular form, while figure 35 has the same information in a bar graph.

Method	Ultimate by Accident Year				
	1995	1996	1997	1998	1999
Paid: Chain Ladder	4,409	5,209	4,813	5,209	6,413
Incurred: Chain Ladder	4,403	5,208	4,809	5,207	6,403
Paid: Bornhuetter-Ferguson	4,410	5,207	4,813	5,204	6,371
Incurred: Bornhuetter-Ferguson	4,404	5,207	4,809	5,204	6,376
Paid: Cape-Cod	4,412	5,210	4,817	5,213	6,390
Incurred: Cape-Cod	4,404	5,208	4,810	5,209	6,389
Paid: Mack Chain Ladder	4,409	5,209	4,813	5,209	6,413
Incurred: Mack Chain Ladder	4,403	5,208	4,809	5,207	6,403
Paid: Munich Chain Ladder	4,409	5,222	4,821	5,220	6,419
Incurred: Munich Chain Ladder	4,403	5,207	4,809	5,207	6,403

Figure 32: Multi-method Development Summary

Method	Ultimate by Accident Year				
	2000	2001	2002	2003	2004
Paid: Chain Ladder	4,392	6,015	7,908	5,655	6,273
Incurred: Chain Ladder	4,411	5,956	8,021	5,815	6,218
Paid: Bornhuetter-Ferguson	4,413	5,906	7,452	5,425	5,454
Incurred: Bornhuetter-Ferguson	4,426	5,876	7,689	5,642	5,708
Paid: Cape-Cod	4,451	5,972	7,561	5,625	5,866
Incurred: Cape-Cod	4,455	5,930	7,769	5,774	5,988
Paid: Mack Chain Ladder	4,392	6,015	7,908	5,655	6,273
Incurred: Mack Chain Ladder	4,411	5,956	8,021	5,815	6,218
Paid: Munich Chain Ladder	4,421	5,974	8,034	5,822	6,236
Incurred: Munich Chain Ladder	4,410	5,959	8,014	5,807	6,220

Figure 33: Multi-method Development Summary

Method	Ultimate by Accident Year		
	2005	2006	Total
Paid: Chain Ladder	4,276	3,786	64,356
Incurred: Chain Ladder	4,758	4,912	66,120
Paid: Bornhuetter-Ferguson	4,760	4,800	64,213
Incurred: Bornhuetter-Ferguson	4,792	4,800	64,934
Paid: Cape-Cod	5,446	5,542	66,504
Incurred: Cape-Cod	5,420	5,576	66,933
Paid: Mack Chain Ladder	4,276	3,786	64,356
Incurred: Mack Chain Ladder	4,758	4,912	66,120
Paid: Munich Chain Ladder	4,733	4,828	66,139
Incurred: Munich Chain Ladder	4,721	4,816	65,976

Figure 34: Multi-method Development Summary

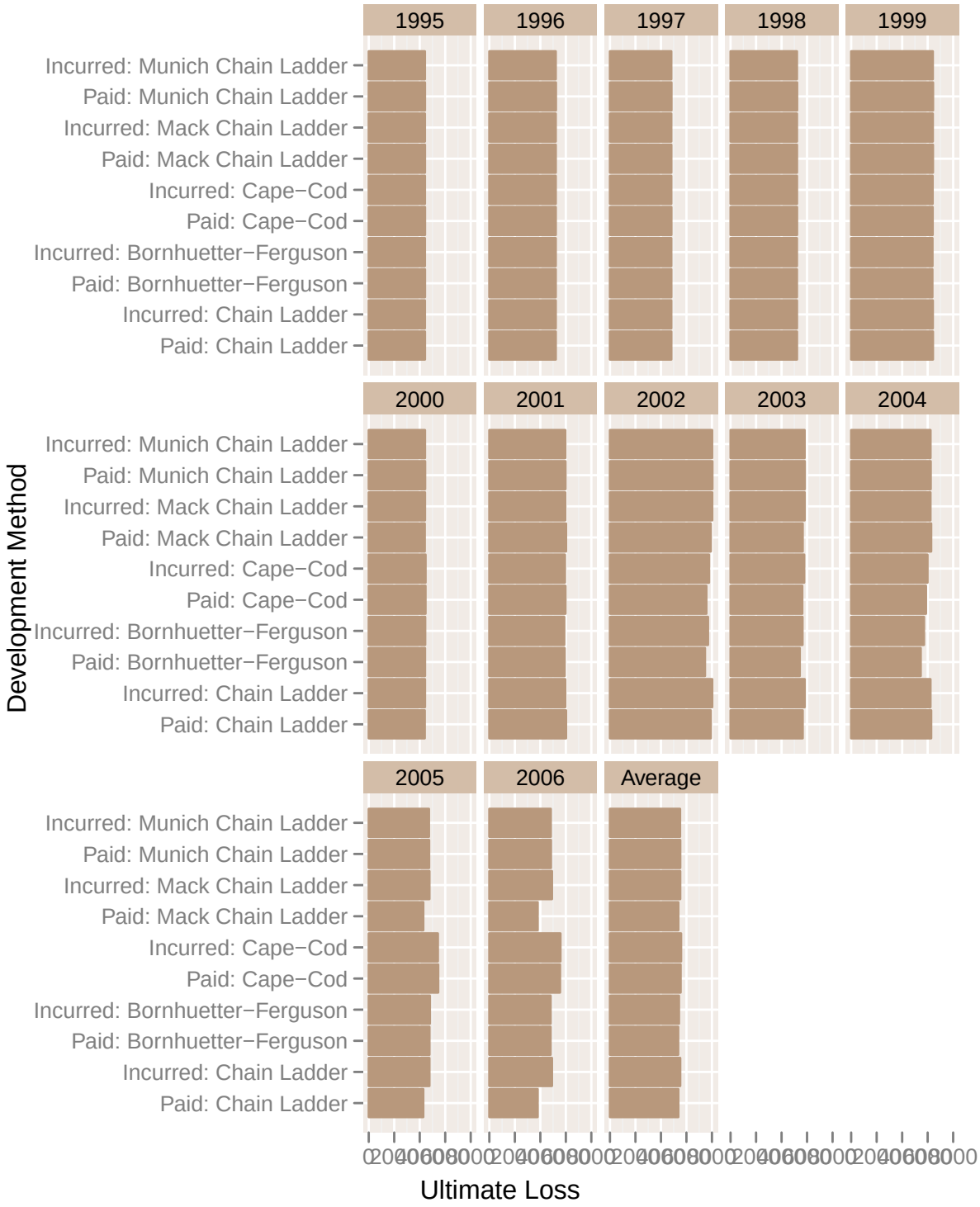

Figure 35: Multi-Method Development Summary Plot

7 Legal

References

[1] G.˜Barnett and B.˜Zehnwirth. Best estimates for reserves. *PCAS*, LXXXVII:245–303, 2000.

[2] E.˜Brosius. Loss development using credibility. *CAS Study Note*, 1993.

[3] J.F. Friedland. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society, 2009.

[4] Thomas Mack. Which stochastic model is underlying the chain ladder method? *http://www.casact.org/pubs/forum/95fforum/95ff229.pdf*, 1993.

[5] Gary˜G. Venter. Testing the assumptions of age-to-age factors. *http://www.casact.org/pubs/proceed/proceed98/980807.pdf*, 1998.