# Basic Reserving: The Chain-Ladder and Additive Loss Methods

By the R Working Party of the CAS

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## Contents



# 1 Introduction

This paper demonstrates in R two classic methods of reserving: the Bornhuetter-Ferguson method (additive loss variant) and the chain-ladder method. Both these methods operate on aggregate loss evaluations in the traditional triangle format. Both these methods work on paid or on case-incurred loss.

We chose these methods because they illustrate the two most basic possible assumptions for a given development period:

**chain-ladder** Developed loss will be a constant multiple of reported loss at the start of the period.

**additive loss** Developed loss will have a constant ratio to premium.

The paper computes reserve ranges for both methods. Bootstrapping, provided by the `ChainLadder` R package by Markus Gesmann, is used for the chain-ladder method. Simple variance assumptions imply ranges for the additive loss method. Both methods assume loss is fully developed by the last development period—neither estimates tail factors or tail uncertainty.

Finally, the paper presents a few graphs and statistics which help evaluate the appropriateness of each model.

# 2 Original Data

Figure 1 contains all the input data required for this paper:

1. a loss triangle (aggregate losses by development age and origin)
2. the corresponding premium or exposure by origin.

Here "origin" refers to the period from which the loss emanates—it could mean accident year, report year, policy year, accident quarter, etc. The loss could be paid or case-incurred, but we will use phrase "reported loss" below. Similarly, the calculations would be the same whether premium or exposure is given; below we will refer to premium and loss ratios for simplicity.

| | | Reported Loss by Development Age | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Origin | Premium | 3 | 15 | 27 | 39 | 51 | 63 | 75 | 87 | 99 | 111 | 123 | 135 |
| 1995 | 6,000 | 44 | 1,331 | 3,319 | 4,020 | 4,232 | 4,252 | 4,334 | 4,369 | 4,386 | 4,395 | 4,401 | 4,399 |
| 1996 | 6,000 | 42 | 1,244 | 3,508 | 4,603 | 4,842 | 4,970 | 5,059 | 5,083 | 5,155 | 5,205 | 5,205 | |
| 1997 | 6,000 | 17 | 1,088 | 3,438 | 4,169 | 4,371 | 4,482 | 4,626 | 4,734 | 4,794 | 4,804 | | |
| 1998 | 6,000 | 10 | 781 | 3,135 | 4,085 | 4,442 | 4,777 | 4,914 | 5,110 | 5,176 | | | |
| 1999 | 6,000 | 13 | 937 | 3,506 | 4,828 | 5,447 | 5,790 | 6,112 | 6,295 | | | | |
| 2000 | 6,000 | 2 | 751 | 2,639 | 3,622 | 3,931 | 4,077 | 4,244 | | | | | |
| 2001 | 6,000 | 4 | 1,286 | 3,570 | 4,915 | 5,377 | 5,546 | | | | | | |
| 2002 | 6,000 | 2 | 911 | 5,023 | 6,617 | 7,194 | | | | | | | |
| 2003 | 6,000 | 3 | 1,398 | 4,021 | 4,825 | | | | | | | | |
| 2004 | 6,000 | 4 | 1,130 | 3,981 | | | | | | | | | |
| 2005 | 6,000 | 21 | 915 | | | | | | | | | | |
| 2006 | 6,000 | 13 | | | | | | | | | | | |

Figure 1: Input Data

# 3 Reserving Results

## 3.1 Bootstrap Chain-Ladder Method

To produce reserve ranges, this paper applies bootstrapping to the traditional chain-ladder method. The algorithm follows England and Verrall and is implemented in the ChainLadder R package by Markus Gesmann. See `http://code.google.com/p/chainladder/` for more information on this package.

| | | Mean Ultimate | | Outstanding Reserves | | | |
|---|---|---|---|---|---|---|---|
| Origin | Latest | Loss | LR (%) | Mean | Std. Dev. | 75th Pct | 95th Pct |
| 1995 | 4,399 | 4,399 | 73.3 | 0 | 0 | 0 | 0 |
| 1996 | 5,205 | 5,202 | 86.7 | -3 | 25 | 0 | 9 |
| 1997 | 4,804 | 4,807 | 80.1 | 3 | 28 | 5 | 62 |
| 1998 | 5,176 | 5,201 | 86.7 | 25 | 57 | 37 | 121 |
| 1999 | 6,295 | 6,391 | 106.5 | 96 | 86 | 140 | 248 |
| 2000 | 4,244 | 4,412 | 73.5 | 168 | 100 | 224 | 360 |
| 2001 | 5,546 | 5,967 | 99.5 | 421 | 154 | 516 | 683 |
| 2002 | 7,194 | 7,996 | 133.3 | 802 | 201 | 939 | 1,102 |
| 2003 | 4,825 | 5,818 | 97.0 | 993 | 261 | 1,140 | 1,426 |
| 2004 | 3,981 | 6,249 | 104.2 | 2,268 | 435 | 2,553 | 3,070 |
| 2005 | 915 | 4,757 | 79.3 | 3,842 | 941 | 4,374 | 5,401 |
| 2006 | 13 | 4,446 | 74.1 | 4,433 | 31,413 | 11,779 | 29,498 |
| Total | 52,597 | 65,646 | 91.2 | 13,049 | 31,421 | 20,008 | 39,052 |

Figure 2: Results of bootstrap chain-ladder

The results of the bootstrap chain-ladder are presented in figure 2. The mean ultimate and reserve amounts should match the traditional loss-weighted chain-ladder method, modulo simulation error. Simulation error for very undeveloped periods (typically the last accident year) may be large, but in practice actuaries rarely use the chain-ladder method in these cases.

The Mack method (see Mack) is an earlier way to estimate reserve uncertainty while preserving the chain-ladder method. It is also implemented in the ChainLadder package and is computationally quicker than bootstrapping. However, the Mack method does not lend itself to percentile calculations, such as the 75th and 95th percentiles shown in figure 2.

## 3.2 Additive Loss Method

The additive loss method is arguably the simplest version of the Bornhuetter-Ferguson method. Unfortunately for such a basic method, it goes under a mess of names: additive loss method, Cape Cod method, Stanard-Buhlmann, incremental loss ratio method, complementary loss ratio method (see Schmidt and Zocher). The additive loss method uses premium estimates by year, but does not require a priori loss ratios. It assumes that the loss developing in each development period is a fixed percentage of premium across all origin years.

For instance, suppose we are trying to reserve for accident year 2008 which is 24 months old. In the past, loss equal to 10% of premium has developed between the ages of 24 and 36 months. Then the additive loss method estimates that 10% of accident year 2008 premium will develop in the next year. It doesn't matter how much accident year 2008 loss has developed so far.

To develop reserve ranges, we will make the very simple assumption that, for a given development period, the incremental loss development for all origin periods will be independent normal distributions with the same mean, with variance inversely proportional to premium. Then we can use simple constant regression with premium weights to estimate the process risk and parameter uncertainty.

The results of the additive loss method are shown in figure 3.

| | | Mean Ultimate | | Outstanding Reserves | | | |
|---|---|---|---|---|---|---|---|
| Origin | Latest | Loss | LR (%) | Mean | Std. Dev. | 75th Pct | 95th Pct |
| 1995 | 4,399 | 4,399 | 73.3 | 0 | 0 | 0 | 0 |
| 1996 | 5,205 | 5,203 | 86.7 | -2 | 2 | -1 | 1 |
| 1997 | 4,804 | 4,805 | 80.1 | 1 | 6 | 5 | 10 |
| 1998 | 5,176 | 5,200 | 86.7 | 24 | 28 | 43 | 69 |
| 1999 | 6,295 | 6,373 | 106.2 | 78 | 39 | 104 | 142 |
| 2000 | 4,244 | 4,431 | 73.8 | 187 | 96 | 252 | 345 |
| 2001 | 5,546 | 5,890 | 98.2 | 344 | 135 | 435 | 565 |
| 2002 | 7,194 | 7,717 | 128.6 | 523 | 185 | 648 | 827 |
| 2003 | 4,825 | 5,720 | 95.3 | 895 | 254 | 1,066 | 1,312 |
| 2004 | 3,981 | 5,934 | 98.9 | 1,953 | 412 | 2,231 | 2,631 |
| 2005 | 915 | 5,396 | 89.9 | 4,481 | 774 | 5,004 | 5,755 |
| 2006 | 13 | 5,550 | 92.5 | 5,537 | 808 | 6,082 | 6,865 |
| Total | 52,597 | 66,617 | 92.5 | 14,020 | 1,245 | 14,860 | 16,068 |

Figure 3: Results of additive loss method

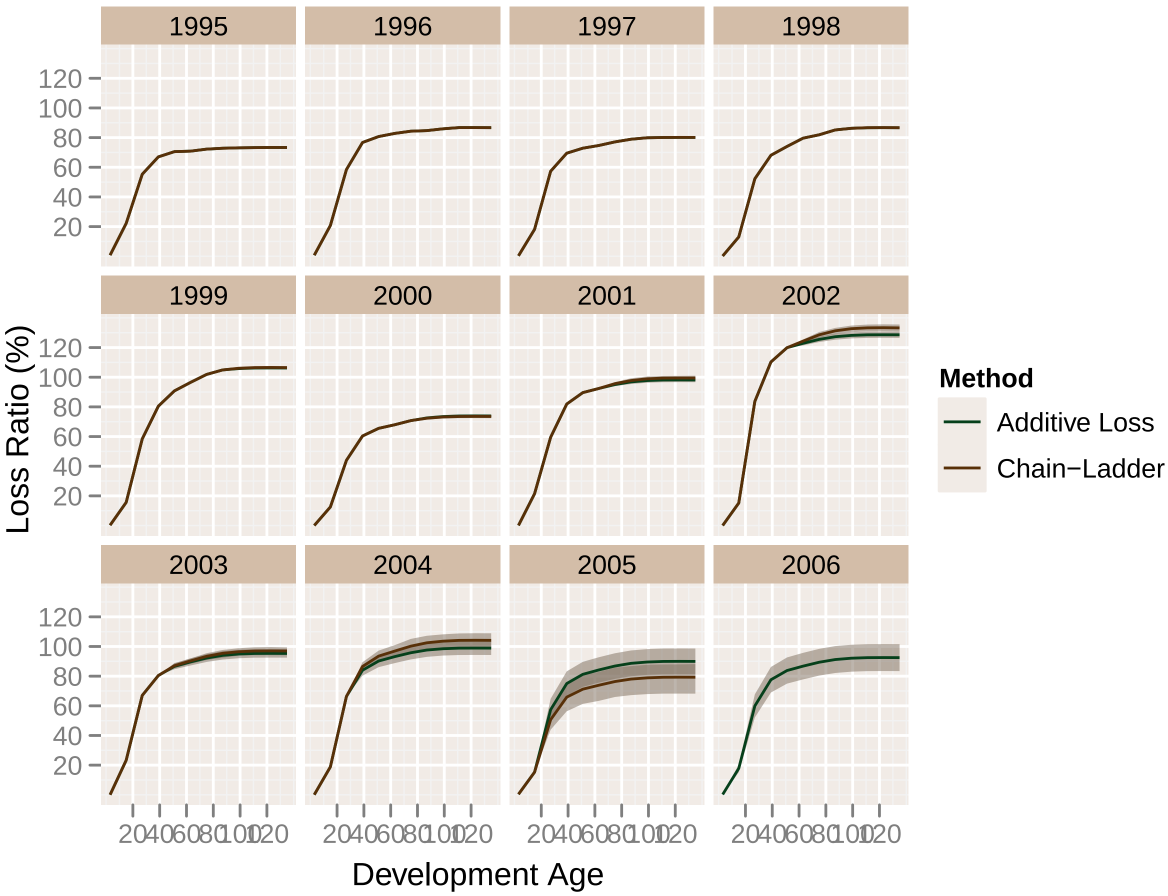


Figure 4: Method Development Comparison by Origin

## 3.3 Comparison Plots

Figure 4 compares the chain-ladder and additive loss projected development by origin. The shaded areas represent the 25th to 75th percentiles for each method.
Figure 5 compares the two methods' distribution of ultimate losses by origin. The histograms represent number of samples from the bootstrap chain-ladder. Our simple additive loss model produces the continuous density curve which has been scaled above to match the histogram.

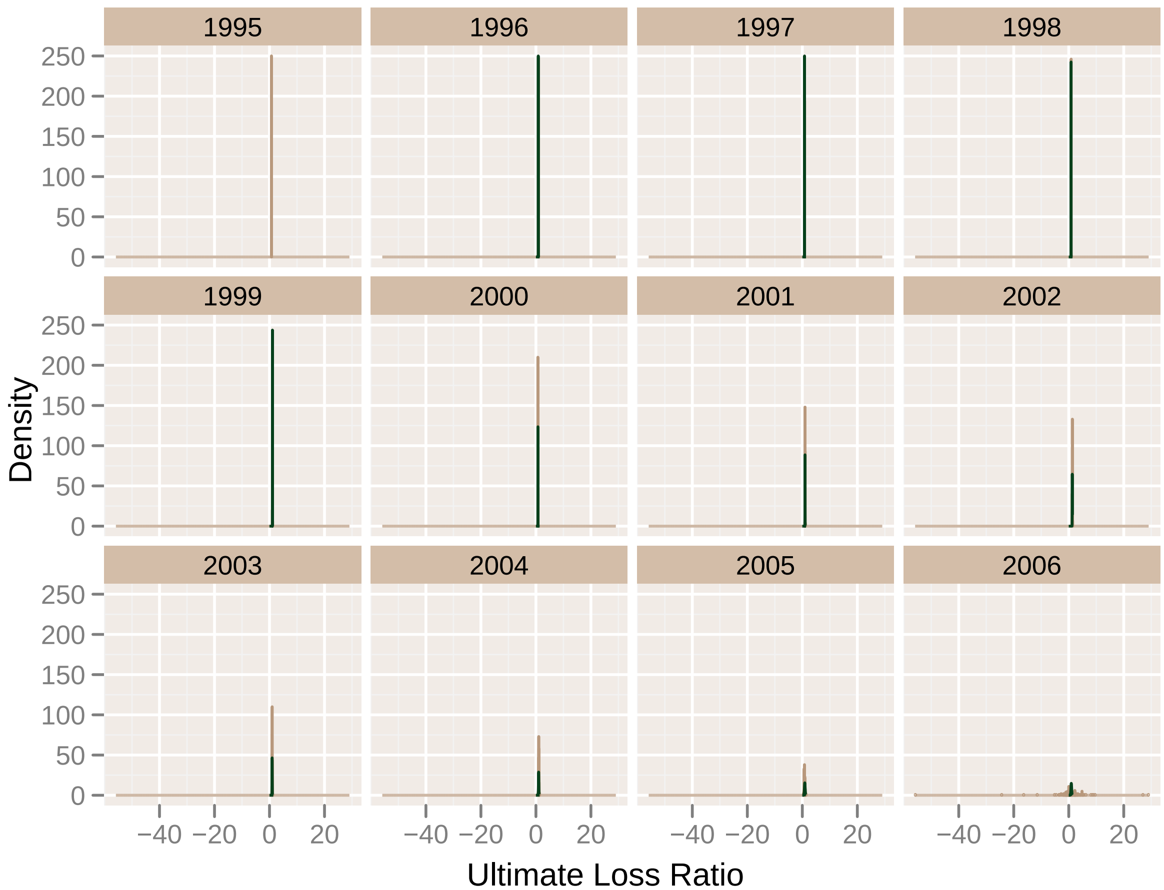


Figure 5: Distribution of Method Ultimates by Origin

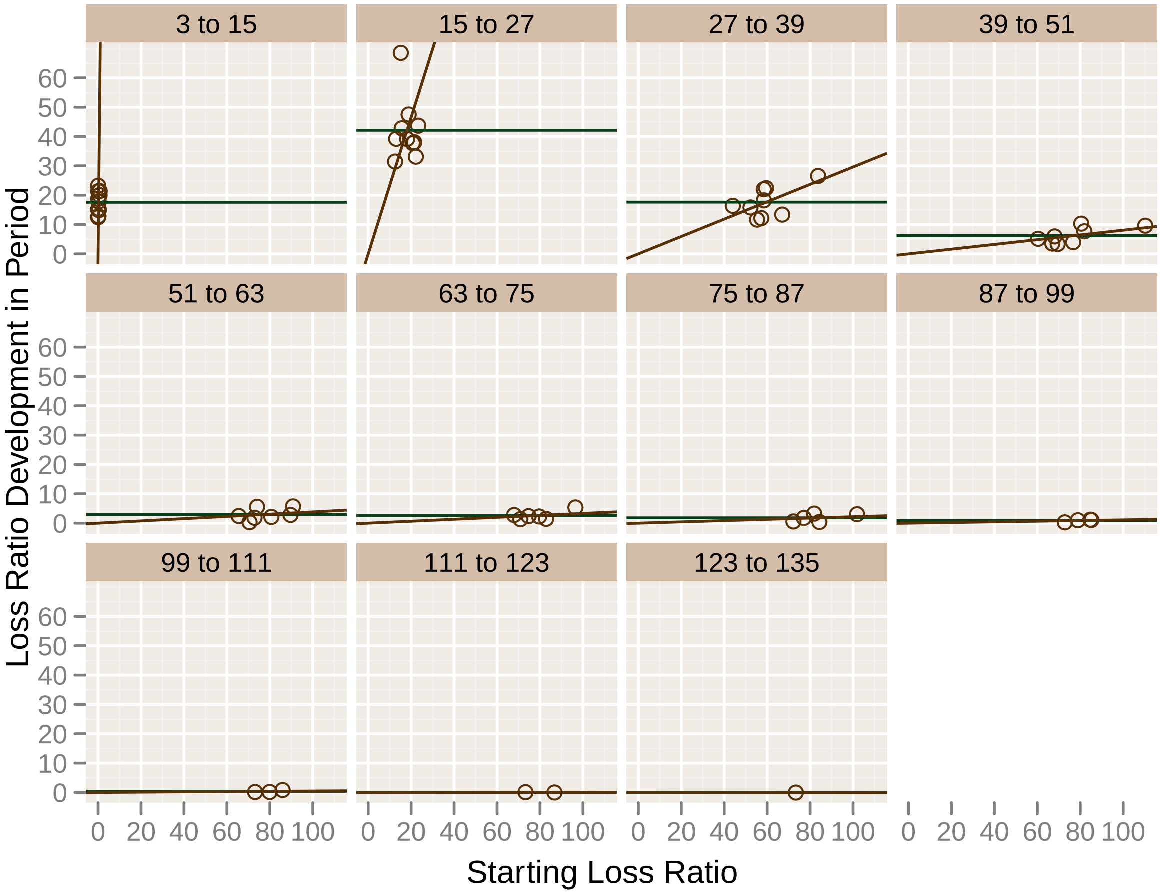


Figure 6: Comparison of Model Fits by Development Period

# 4 Graphical Diagnostics

Graphs are popular for evaluating the appropriateness of a stochastic reserving model (see Barnett and Zehnwirth, Brosius, and Venter for more information on their use as reserving diagnostics).

Figure 6 plots incremental loss ratio vs starting loss ratio for each development period. If the chain-ladder model worked perfectly, the incremental loss ratio would be proportional to the starting loss ratio; all the points would fall on a line going through the origin. If the Bornhuetter-Ferguson method worked perfectly, the incremental loss ratio would be independent of starting loss ratio; all the points would fall on a horizontal line. Thus graph like figure 6 is a simple way to visually judge which if either method is working.

Residuals are another way to judge the appropriateness of a model. A model's residual is an actual observed value minus the models predicted value. Figure 7 shows residuals by development period, loss ratio, origin, and calendar period. The smoothing line is produced by local polynomial regression and may aid the reader in quickly picking up trends.

If the residuals show a trend, it's a warning sign that the reserving method may be inappropriate. For instance, if a company's claims department strengthens its case reserving, it might cause a method to overpredict development (have a negative residual) for those calendar periods. To take another example, if rate adequacy is slipping, then the additive loss method might show a positive trend in the residuals by origin, but the chain-ladder method may continue to fit well.

Assuming displays such as figures 6 and 7 can be produced automatically (as in R), they give actuaries a quick and effective way to evaluate reserving methods.

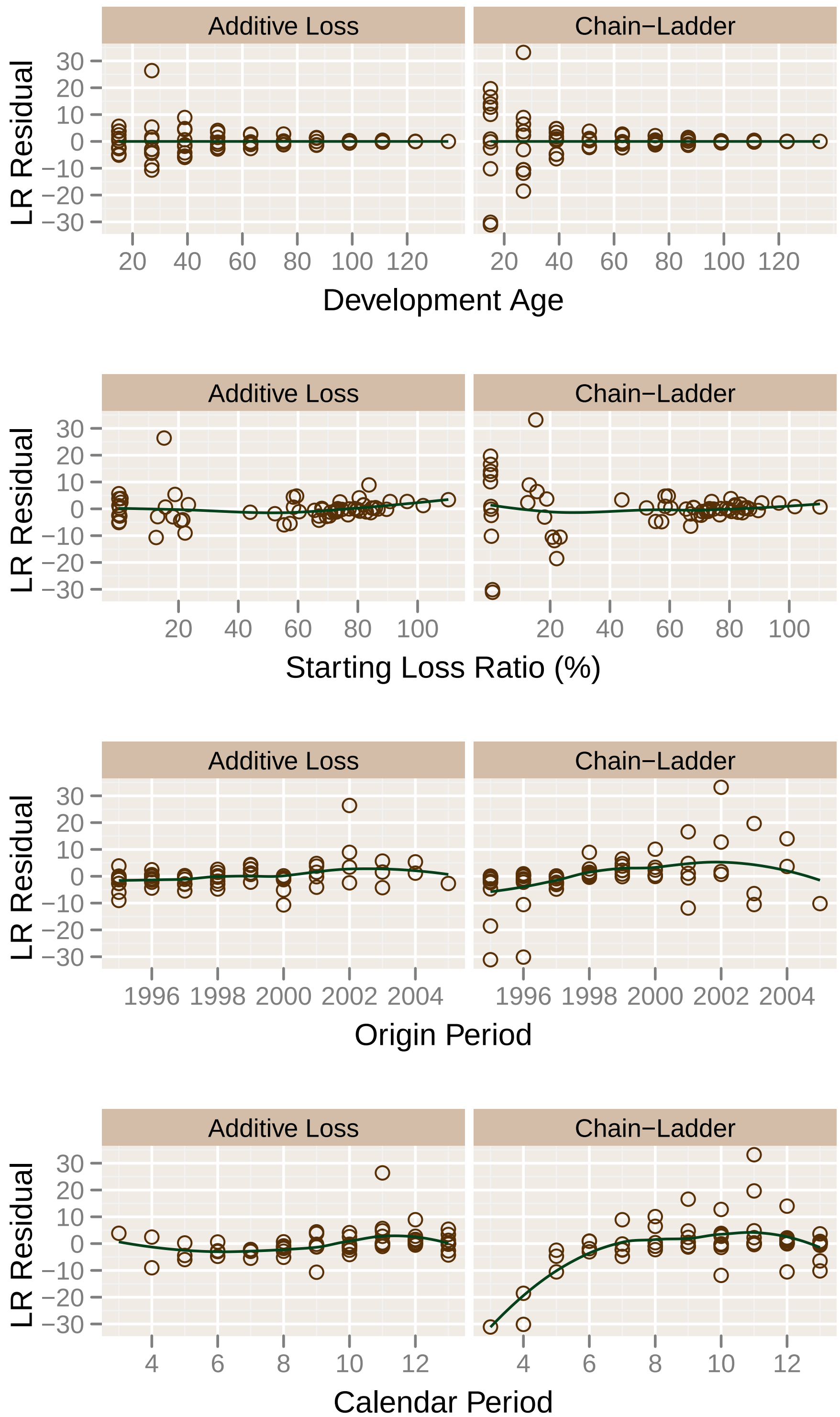


Figure 7: Comparison of Residuals

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# 6 Legal