On the usage of the `pbkrtest` package
* WORKING DOCUMENT **

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Contents

1 Introduction

The `shoes` data is a list of two vectors, giving the wear of shoes of materials A and B for one foot each of ten boys.

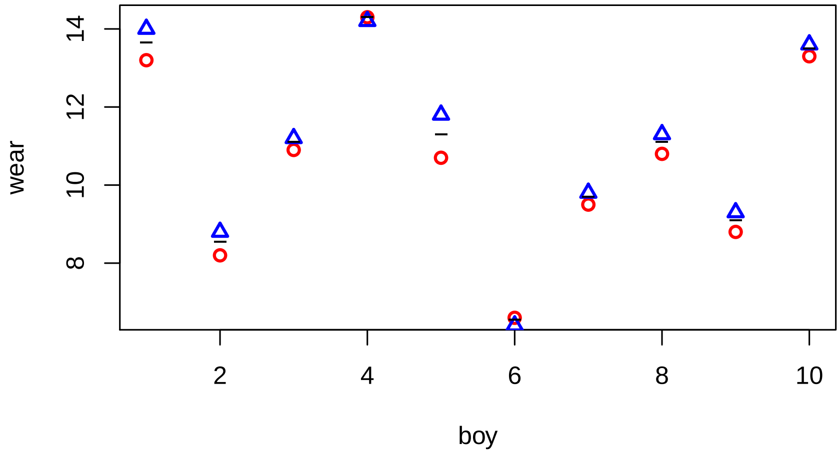
```
R> data(shoes, package="MASS")
R> shoes

$A
 [1] 13.2  8.2 10.9 14.3 10.7  6.6  9.5 10.8  8.8 13.3

$B
 [1] 14.0  8.8 11.2 14.2 11.8  6.4  9.8 11.3  9.3 13.6
```

A plot clearly reveals that boys wear their shoes differently.

```
R> plot(A~1, data=shoes, col='red',lwd=2, pch=1, ylab="wear", xlab="boy")
R> points(B~1, data=shoes, col='blue',lwd=2,pch=2)
R> points(I((A+B)/2)~1, data=shoes, pch='-', lwd=2)
```



One option for testing the effect of materials is to make a paired t–test. The following forms are equivalent:

```
R> r1<-t.test(shoes$A, shoes$B, paired=T)
R> r2<-t.test(shoes$A-shoes$B)
R> r1

        Paired t-test

data:  shoes$A and shoes$B
t = -3.3489, df = 9, p-value = 0.008539
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.6869539 -0.1330461
sample estimates:
mean of the differences
                  -0.41
```

To work with data in a mixed model setting we create a dataframe:

```
R> boy <- rep(1:10,2)
R> boyf<- factor(letters[boy])
R> mat <- factor(c(rep("A", 10), rep("B",10)))
R> shoedf <- data.frame(wear=unlist(shoes), boy=boy, boyf=boyf, mat=mat)
R> head(shoedf)

   wear boy boyf mat
A1 13.2   1    a   A
A2  8.2   2    b   A
A3 10.9   3    c   A
A4 14.3   4    d   A
A5 10.7   5    e   A
A6  6.6   6    f   A
```

For later use we create an imbalanced version of data:

```
R> shoedf2 <-  shoedf[-c(9,12),]
```

```
R> lmm1 <- lmer(wear~mat+(1|boyf), data=shoedf)
R> lmm0 <- update(lmm1, .~.-mat)
R> lmm1i <- lmer(wear~mat+(1|boyf), data=shoedf2)
R> lmm0i <- update(lmm1i, .~.-mat)
```

The asymptotic likelihood ratio test shows stronger significance than the t–test:

```
R> anova(lmm1, lmm0, test="Chisq")

Data: shoedf
Models:
lmm0: wear ~ (1 | boyf)
lmm1: wear ~ mat + (1 | boyf)
     Df    AIC    BIC  logLik  Chisq Chi Df Pr(>Chisq)
lmm0  3 67.937 70.924 -30.968
lmm1  4 61.817 65.800 -26.909 8.1197      1   0.004379 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R> anova(lmm1i, lmm0i, test="Chisq")

Data: shoedf2
Models:
lmm0i: wear ~ (1 | boyf)
lmm1i: wear ~ mat + (1 | boyf)
      Df    AIC    BIC  logLik  Chisq Chi Df Pr(>Chisq)
lmm0i  3 64.757 67.428 -29.378
lmm1i  4 61.668 65.230 -26.834 5.0883      1    0.02409 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

2 Kenward–Roger approach

The Kenward–Roger approximation is exact in this case

```
R> (kr<-KRmodcomp(lmm1, lmm0))

F-test with Kenward-Roger approximation; computing time: 0.14 sec.
large : wear ~ mat + (1 | boyf)
small : wear ~ (1 | boyf)
        stat    ndf    ddf F.scaling  p.value
Ftest 11.215  1.000  9.000         1 0.008539 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
R> summary(kr)

F-test with Kenward-Roger approximation; computing time: 0.14 sec.
large : wear ~ mat + (1 | boyf)
small : wear ~ (1 | boyf)
         stat    ndf    ddf F.scaling  p.value
Ftest  11.215  1.000  9.000         1 0.008539 **
FtestU 11.215  1.000  9.000           0.008539 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Relevant information can be retrieved with

```
R> getKR(kr, "ddf")

[1] 9
```

```
R> (kri<-KRmodcomp(lmm1i, lmm0i))

F-test with Kenward-Roger approximation; computing time: 0.06 sec.
large : wear ~ mat + (1 | boyf)
small : wear ~ (1 | boyf)
        stat    ndf    ddf F.scaling p.value
Ftest 6.1287 1.0000 7.0260         1 0.04236 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

3 Parametric bootstrap

Parametric bootstrap provides an alternative but many simulations are often needed to provide credible results:

```
R> (pb<-PBmodcomp(lmm1, lmm0, nsim=99))

PBmodcomp.lmerMod
Parametric bootstrap test;  computing time: 1.24 sec.
Requested samples: 99 Used samples: 99 Extremes: 3
large : wear ~ mat + (1 | boyf)
small : wear ~ (1 | boyf)
         stat df  p.value
LRT    8.1197  1 0.004379 **
PBtest 8.1197    0.040000 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
R> summary(pb)

Parametric bootstrap test;  computing time: 1.24 sec.
Requested samples: 99 Used samples: 99 Extremes: 3
large : wear ~ mat + (1 | boyf)
small : wear ~ (1 | boyf)
           stat     df    ddf  p.value
LRT      8.1197 1.0000        0.004379 **
PBtest   8.1197               0.040000 *
Gamma    8.1197               0.019163 *
F        8.1197 1.0000 6.9554 0.024874 *
Bartlett 5.7849 1.0000        0.016164 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
R> (pbi<-PBmodcomp(lmm1i, lmm0i, nsim=99))

PBmodcomp.lmerMod
Parametric bootstrap test;  computing time: 1.22 sec.
Requested samples: 99 Used samples: 99 Extremes: 7
large : wear ~ mat + (1 | boyf)
small : wear ~ (1 | boyf)
         stat df p.value
LRT    5.0883  1 0.02409 *
PBtest 5.0883    0.08000 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4 With linear models

```
R> lm1<-lm(wear~mat+boyf, data=shoedf)
R> lm0<-update(lm1, .~.-mat)
R> anova(lm1, lm0)

Analysis of Variance Table

Model 1: wear ~ mat + boyf
Model 2: wear ~ boyf
  Res.Df    RSS Df Sum of Sq      F   Pr(>F)
1      9 0.6745
2     10 1.5150 -1   -0.8405 11.215 0.008539 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
R> lm1i<-lm(wear~mat+boyf, data=shoedf2)
R> lm0i<-update(lm1i, .~.-mat)
R> anova(lm1i, lm0i)

Analysis of Variance Table

Model 1: wear ~ mat + boyf
Model 2: wear ~ boyf
  Res.Df    RSS Df Sum of Sq      F  Pr(>F)
1      7 0.6475
2      8 1.2100 -1   -0.5625 6.0811 0.04309 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

A Matrices for random effects

The matrices involved in the random effects can be obtained with

```
R> shoedf3 <- subset(shoedf, boy<=5)
R> lmm1 <- lmer(wear~mat+(1|boyf), data=shoedf3)
R> SG <- LMM_Sigma_G(lmm1)
```

```
R> round(SG$Sigma*10)

10 x 10 sparse Matrix of class "dgCMatrix"

 [1,] 53  .  .  .  . 52  .  .  .  .
 [2,]  . 53  .  .  .  . 52  .  .  .
 [3,]  .  . 53  .  .  .  . 52  .  .
 [4,]  .  .  . 53  .  .  .  . 52  .
 [5,]  .  .  .  . 53  .  .  .  . 52
 [6,] 52  .  .  .  . 53  .  .  .  .
 [7,]  . 52  .  .  .  . 53  .  .  .
 [8,]  .  . 52  .  .  .  . 53  .  .
 [9,]  .  .  . 52  .  .  .  . 53  .
[10,]  .  .  .  . 52  .  .  .  . 53
```

```
R> SG$G

[[1]]
10 x 10 sparse Matrix of class "dgCMatrix"

 [1,] 1 . . . . 1 . . . .
 [2,] . 1 . . . . 1 . . .
 [3,] . . 1 . . . . 1 . .
 [4,] . . . 1 . . . . 1 .
 [5,] . . . . 1 . . . . 1
 [6,] 1 . . . . 1 . . . .
 [7,] . 1 . . . . 1 . . .
 [8,] . . 1 . . . . 1 . .
 [9,] . . . 1 . . . . 1 .
[10,] . . . . 1 . . . . 1

[[2]]
10 x 10 sparse Matrix of class "dgCMatrix"

 [1,] 1 . . . . . . . . .
 [2,] . 1 . . . . . . . .
 [3,] . . 1 . . . . . . .
 [4,] . . . 1 . . . . . .
 [5,] . . . . 1 . . . . .
 [6,] . . . . . 1 . . . .
 [7,] . . . . . . 1 . . .
 [8,] . . . . . . . 1 . .
 [9,] . . . . . . . . 1 .
[10,] . . . . . . . . . 1
```