Package ‘demofit’

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Title Parametric Mortality Curve Fitting and Mortality Forecasting Tools

Version 0.1.2

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Description Provides tools for fitting parametric mortality curves. Implements multiple optimisation strategies to enhance robustness and stability of parameter estimation. Offers tools for forecasting mortality rates guided by mortality curves.
For modelling details see:
Tabeau (2001) <doi:10.1007/0-306-47562-6_1>,
Renshaw and Haberman (2006) <doi:10.1016/j.insmatheco.2005.12.001>,
Cairns et al. (2009) <doi:10.1080/10920277.2009.10597538>.

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Imports forecast, minpack.lm, MortalityLaws, NlcOptim, stats, graphics, grDevices

NeedsCompilation no

Repository CRAN

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Contents

APCS *Age-period-cohort model*

Description

Fits and forecasts mortality rates using age-period-cohort model.

Usage

```
APCS(
  x,
  M,
 curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
   "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The age-period-cohort (APC) model is specified as

$ln(m_{x,t}) = \alpha_x + \kappa_t + \gamma_{t-x} + \epsilon_{x,t}$.

The model is estimated by Newton updating scheme and is forecasted by ARIMA applied to κ_t and γ_c. Constraints include sum of κ_t is zero and and sum of γ_c is zero. It can be applied to whole age range.

Value

An object of class APCS with associated S3 methods coef, forecast, plot, and residuals.

References

Bray, I. (2002). Application of Markov chain Monte Carlo methods to projecting cancer incidence and mortality. Journal of the Royal Statistical Society Series C, 51(2), 151-164.

Examples

```
x <- 60:89
a <- c(-4.8499,-4.7676,-4.6719,-4.5722,-4.4847,-4.3841,-4.2813,-4.1863,-4.0861,-3.9962,
-3.8885,-3.7896,-3.6853,-3.5737,-3.4728,-3.3718,-3.2586,-3.1474,-3.0371,-2.9206,
-2.7998,-2.6845,-2.5653,-2.4581,-2.3367,-2.2159,-2.1017,-1.9941,-1.8821, -1.7697)
b <- rep(1/30,30)
k <- c(12.11,10.69,11.18,9.64,9.35,8.21,6.89,5.74,4.56,3.6,
3.27,2.04,1.11,-0.44,-1.05,-1.03,-1.84,-2.9,-4.03,-4.12,
-5.18,-5.64,-6,-6.51,-6.91,-6.9,-8.32,-8.53,-9.69,-9.31)
set.seed(123)
M <- exp(outer(k,b)+matrix(a,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.035))
fit <- APCS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

CBDCS *CBD with cohort model*

Description

Fits and forecasts mortality rates using CBD with cohort model.

Usage

```
CBDCS(
  x,
  M,
  curve = c("gompertz", "makeham", "perks", "weibull", "beard", "martinelle", "thatcher",
```

```
    "gompertz2", "makeham2", "perks2", "weibull2", "beard2", "martinelle2", "thatcher2"),
   h = 10,
   jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, perks, weibull, beard, martinelle, thatcher, gompertz2, makeham2, perks2, weibull2, beard2, martinelle2, thatcher2, where first 7 curves' parameters are unconstrained and last 7 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The CBD with cohort (M6) model is specified as

$ln(m_{x,t}) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}) + \gamma_{t-x} + \epsilon_{x,t}$.

The model is estimated by Newton updating scheme and is forecasted by ARIMA applied to $\kappa_{1,t}$, $\kappa_{2,t}$, and γ_c. Constraints include sum of γ_c is zero and sum of $c\gamma_c$ is zero. It is designed for ages 50-90.

Value

An object of class CBDCS with associated S3 methods coef, forecast, plot, and residuals.

References

Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Balevich, I. (2009). A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. North American Actuarial Journal, 13(1), 1-35.

Examples

```
x <- 60:89
k1 <- -2.97-0.0245*(0:29)
k2 <- 0.101+0.000345*(0:29)
set.seed(123)
M <- exp(matrix(k1,nrow=30,ncol=30,byrow=FALSE)+outer(k2,(x-mean(x)))+rnorm(900,0,0.035))
fit <- CBDCS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

`CBDQCS` *CBD with curvature and cohort model*

Description

Fits and forecasts mortality rates using CBD with curvature and cohort model.

Usage

```
CBDQCS(
  x,
  M,
  curve = c("gompertz", "makeham", "perks", "weibull", "beard", "martinelle", "thatcher",
   "gompertz2", "makeham2", "perks2", "weibull2", "beard2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, perks, weibull, beard, martinelle, thatcher, gompertz2, makeham2, perks2, weibull2, beard2, martinelle2, thatcher2, where first 7 curves' parameters are unconstrained and last 7 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The CBD with curvature and cohort (M7) model is specified as

$ln(m_{x,t}) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}) + \kappa_{3,t}((x - \bar{x})^2 - \sigma^2) + \gamma_{t-x} + \epsilon_{x,t}$.

The model is estimated by Newton updating scheme and is forecasted by ARIMA applied to $\kappa_{1,t}$, $\kappa_{2,t}$, $\kappa_{3,t}$, and γ_c. Constraints include sum of γ_c is zero, sum of $c\gamma_c$ is zero, and sum of $c^2\gamma_c$ is zero. It is designed for ages 50-90.

Value

An object of class CBDQCS with associated S3 methods coef, forecast, plot, and residuals.

References

Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Balevich, I. (2009). A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. North American Actuarial Journal, 13(1), 1-35.

Examples

```
x <- 60:89
k1 <- -2.97-0.0245*(0:29)
k2 <- 0.101+0.000345*(0:29)
set.seed(123)
M <- exp(matrix(k1,nrow=30,ncol=30,byrow=FALSE)+outer(k2,(x-mean(x)))+rnorm(900,0,0.035))
fit <- CBDQCS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

`CBDS` *CBD model*

Description

Fits and forecasts mortality rates using CBD model.

Usage

```
CBDS(
  x,
  M,
 curve = c("gompertz", "makeham", "perks", "weibull", "beard", "martinelle", "thatcher",
   "gompertz2", "makeham2", "perks2", "weibull2", "beard2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, perks, weibull, beard, martinelle, thatcher, gompertz2, makeham2, perks2, weibull2, beard2, martinelle2, thatcher2, where first 7 curves' parameters are unconstrained and last 7 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The CBD (M5) model is specified as

$$ln(m_{x,t}) = \kappa_{1,t} + \kappa_{2,t}(x - \bar{x}) + \epsilon_{x,t}.$$

The model is estimated by regression and is forecasted by ARIMA applied to $\kappa_{1,t}$ and $\kappa_{2,t}$. It is designed for ages 50-90.

Value

An object of class CBDS with associated S3 methods coef, forecast, plot, and residuals.

References

Cairns, A.J.G., Blake, D., and Dowd, K. (2006). A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration. Journal of Risk and Insurance, 73(4), 687-718.

Examples

```
x <- 60:89
k1 <- -2.97-0.0245*(0:29)
k2 <- 0.101+0.000345*(0:29)
set.seed(123)
M <- exp(matrix(k1,nrow=30,ncol=30,byrow=FALSE)+outer(k2,(x-mean(x)))+rnorm(900,0,0.035))
fit <- CBDS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

CC *Mortality curve calculation*

Description

Calculates mortality rates given mortality curve parameter values.

Usage

```
CC(
  x,
  par,
  curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
    "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
     "martinelle", "thatcher")
)
```

Arguments

- `x` — vector of ages.
- `par` — vector of mortality curve parameter values.
- `curve` — name of mortality curve (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher).

Details

See `MC()` for more details of different mortality curves.

Value

Calculated mortality rates based on selected mortality curve and parameter values.

Examples

```
CC(x=60:89,par=c(0.0000082,0.10771),curve="gompertz")
```

`CFMS` *Common factor model*

Description

Fits and forecasts mortality rates of two populations using common factor model.

Usage

```
CFMS(
  x,
  M1,
  M2,
  curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
    "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M1`	matrix of mortality rates of population 1 (rows as years and columns as ages).
`M2`	matrix of mortality rates of population 2 (rows as years and columns as ages).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The common factor model (CFM) is specified as

$ln(m_{x,t,i}) = \alpha_{x,i} + B_x K_t + \beta_{x,i}\kappa_{t,i} + \epsilon_{x,t,i}$.

The model is estimated by Newton updating scheme and is forecasted by ARIMA applied to K_t and $\kappa_{t,i}$. Constraints include sum of B_x is one, sum of K_t is zero, sum of $\beta_{x,i}$ is one, and sum of $\kappa_{t,i}$ is zero. It can be applied to whole age range.

Value

An object of class CFMS with associated S3 methods coef, forecast, plot (which = 1 gives parameter estimates; which = 2 gives residuals and forecasts), and residuals.

References

Li, N. and Lee, R. (2005). Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. Demography, 42(3), 575-594.

Examples

```
x <- 60:89
a1 <- c(-5.18,-5.12,-4.98,-4.92,-4.82,-4.73,-4.66,-4.53,-4.45,-4.35,
-4.26,-4.17,-4.05,-3.95,-3.84,-3.73,-3.65,-3.52,-3.40,-3.29,
-3.14,-3.02,-2.88,-2.76,-2.64,-2.49,-2.37,-2.25,-2.12,-2.00)
a2 <- c(-4.78,-4.68,-4.57,-4.49,-4.39,-4.29,-4.19,-4.10,-4.00,-3.89,
-3.80,-3.69,-3.60,-3.49,-3.39,-3.29,-3.17,-3.07,-2.96,-2.85,
-2.71,-2.62,-2.49,-2.37,-2.26,-2.14,-2.04,-1.91,-1.82,-1.72)
B <- c(0.0381,0.0340,0.0420,0.0389,0.0423,0.0414,0.0406,0.0393,0.0415,0.0400,
0.0411,0.0362,0.0387,0.0381,0.0384,0.0385,0.0356,0.0314,0.0317,0.0337,
0.0316,0.0298,0.0284,0.0270,0.0248,0.0262,0.0205,0.0215,0.0142,0.0145)
K <- c(9.66,9.89,10.66,9.83,9.52,7.39,7.64,6.36,2.32,4.18,
2.91,-0.61,0.28,-0.38,-1.79,-3.34,-1.74,-3.50,-4.28,-4.77,
-4.98,-7.13,-5.09,-6.41,-5.56,-5.65,-6.12,-5.64,-7.35,-6.28)
```

```
b1 <- c(0.0012,-0.0033,0.0523,0.0161,0.0529,0.0220,0.0312,0.0437,0.0709,0.0444,
0.0398,0.0361,0.0403,0.0396,0.0506,0.0315,0.0428,0.0261,0.0384,0.0388,
0.0300,0.0269,0.0275,0.0256,0.0239,0.0421,0.0314,0.0284,0.0174,0.0314)
k1 <- c(-1.24,-1.38,-3.48,-2.51,-1.32,-1.90,-3.42,-0.94,0.24,-0.48,
-0.26,2.70,1.39,-0.46,1.74,2.53,0.90,1.43,0.76,2.48,
0.74,2.32,0.42,1.69,-0.64,1.30,0.19,-0.69,-1.11,-1.01)
b2 <- c(-0.0014,0.0272,0.0083,0.0273,0.0209,0.0253,0.0144,0.0333,0.0460,0.0439,
0.0439,0.0674,0.0331,0.0443,0.0312,0.0240,0.0570,0.0312,0.0403,0.0376,
0.0500,0.0289,0.0466,0.0418,0.0349,0.0149,0.0366,0.0178,0.0361,0.0372)
k2 <- c(2.35,0.62,-0.38,0.12,0.00,0.80,-1.39,0.38,2.47,0.40,
0.76,3.06,1.42,-0.73,0.79,1.94,0.12,0.60,-0.43,0.29,
0.17,0.98,-1.01,-0.13,-2.46,-1.24,-1.65,-2.48,-2.32,-3.06)
set.seed(123)
M1 <- exp(outer(k1,b1)+outer(K,B)+matrix(a1,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.07))
M2 <- exp(outer(k2,b2)+outer(K,B)+matrix(a2,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.07))
fit <- CFMS(x=x,M1=M1,M2=M2,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

`curveS3` *Common S3 methods for mortality curves*

Description

Common S3 methods for objects returned by `MC()`:

coef(object) gives estimated parameter values.

fitted(object) gives fitted values.

predict(object,x) gives estimated mortality rate(s) for given age(s) x.

plot(object) plots data and fitted mortality curve.

deviance(object) gives weighted sum of squared residuals on $ln(m_x)$.

residuals(object) gives residuals on $ln(m_x)$.

`ENS` *Mortality ensemble*

Description

Fits and forecasts mortality rates using mortality ensemble.

Usage

```
ENS(
  x,
  M,
  wm = rep(1/7, 7),
 curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
   "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`wm`	vector of weights for LC, RH, APC, M5, M6, M7, and STAR models (default = 1/7).
`curve`	name of mortality curve for smoothing ensemble mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

Ensemble forecast is obtained as a weighted average of forecasts from individual stochastic mortality models. See `LCS()`, `RHS()`, `APCS()`, `CBDS()`, `CBDCS()`, `CBDQCS()`, and `STARS()` for more details of different stochastic mortality models.

Value

An object of class ENS with associated S3 methods forecast and plot.

Examples

```
x <- 60:69
a <- c(-4.8499,-4.7676,-4.6719,-4.5722,-4.4847,-4.3841,-4.2813,-4.1863,-4.0861,-3.9962)
b <- c(0.0801,0.0909,0.0948,0.0951,0.0965,0.1014,0.1042,0.1141,0.1110,0.1118)
k <- c(12.11,10.69,11.18,9.64,9.35,8.21,6.89,5.74,4.56,3.60,
3.27,2.04,1.11,-0.44,-1.05,-1.03,-1.84,-2.90,-4.03,-4.12,
```

```
-5.18,-5.64,-6.00,-6.51,-6.91,-6.90,-8.32,-8.53,-9.69,-9.31)
set.seed(123)
M <- exp(outer(k,b)+matrix(a,nrow=30,ncol=10,byrow=TRUE)+rnorm(300,0,0.035))
fit <- ENS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
forecast::forecast(fit)
plot(fit)
```

FCS *Mortality model fitting*

Description

Fits and forecasts mortality rates using different stochastic mortality models with different estimation methods.

Usage

```
FCS(
  x,
  M,
  model = c("LC", "RH", "APC", "CBD", "CBDC", "CBDQC", "STAR"),
 curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
   "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`model`	name of stochastic mortality model (including LC, RH, APC, CBD, CBDC, CBDQC, and STAR).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

See `LCS()`, `RHS()`, `APCS()`, `CBDS()`, `CBDCS()`, `CBDQCS()`, and `STARS()` for more details of different stochastic mortality models.

Value

An object of class based on selected stochastic mortality model with associated S3 methods coef, forecast, plot, and residuals.

Examples

```
x <- 60:89
a <- c(-4.8499,-4.7676,-4.6719,-4.5722,-4.4847,-4.3841,-4.2813,-4.1863,-4.0861,-3.9962,
-3.8885,-3.7896,-3.6853,-3.5737,-3.4728,-3.3718,-3.2586,-3.1474,-3.0371,-2.9206,
-2.7998,-2.6845,-2.5653,-2.4581,-2.3367,-2.2159,-2.1017,-1.9941,-1.8821, -1.7697)
b <- c(0.0283,0.0321,0.0335,0.0336,0.0341,0.0358,0.0368,0.0403,0.0392,0.0395,
0.0396,0.0399,0.0397,0.0386,0.039,0.0375,0.0367,0.0368,0.035,0.0354,
0.0336,0.0323,0.0313,0.0295,0.0282,0.0265,0.024,0.0226,0.0219,0.0183)
k <- c(12.11,10.69,11.18,9.64,9.35,8.21,6.89,5.74,4.56,3.6,
3.27,2.04,1.11,-0.44,-1.05,-1.03,-1.84,-2.9,-4.03,-4.12,
-5.18,-5.64,-6,-6.51,-6.91,-6.9,-8.32,-8.53,-9.69,-9.31)
set.seed(123)
M <- exp(outer(k,b)+matrix(a,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.035))
fit <- FCS(x=x,M=M,model="LC",curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

`LCS` *Lee-Carter model*

Description

Fits and forecasts mortality rates using Lee-Carter model.

Usage

```
LCS(
  x,
  M,
  curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
    "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

- `x` vector of ages.
- `M` matrix of mortality rates (rows as years and columns as ages).
- `curve` name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
- `h` forecast horizon (default = 10).
- `jumpoff` if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The Lee-Carter (LC) model is specified as

$ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \epsilon_{x,t}$.

The model is estimated by singular value decomposition and is forecasted by ARIMA applied to κ_t. Constraints include sum of β_x is one and sum of κ_t is zero. It can be applied to whole age range.

Value

An object of class LCS with associated S3 methods coef, forecast, plot, and residuals.

References

Lee, R.D. and Carter, L.R. (1992). Modeling and forecasting U.S. mortality. Journal of the American Statistical Association, 87(419), 659-671.

Examples

```
x <- 60:89
a <- c(-4.8499,-4.7676,-4.6719,-4.5722,-4.4847,-4.3841,-4.2813,-4.1863,-4.0861,-3.9962,
-3.8885,-3.7896,-3.6853,-3.5737,-3.4728,-3.3718,-3.2586,-3.1474,-3.0371,-2.9206,
-2.7998,-2.6845,-2.5653,-2.4581,-2.3367,-2.2159,-2.1017,-1.9941,-1.8821, -1.7697)
b <- c(0.0283,0.0321,0.0335,0.0336,0.0341,0.0358,0.0368,0.0403,0.0392,0.0395,
0.0396,0.0399,0.0397,0.0386,0.039,0.0375,0.0367,0.0368,0.035,0.0354,
0.0336,0.0323,0.0313,0.0295,0.0282,0.0265,0.024,0.0226,0.0219,0.0183)
k <- c(12.11,10.69,11.18,9.64,9.35,8.21,6.89,5.74,4.56,3.6,
3.27,2.04,1.11,-0.44,-1.05,-1.03,-1.84,-2.9,-4.03,-4.12,
-5.18,-5.64,-6,-6.51,-6.91,-6.9,-8.32,-8.53,-9.69,-9.31)
set.seed(123)
M <- exp(outer(k,b)+matrix(a,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.035))
fit <- LCS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
```

```
residuals(fit)
```

MC *Mortality curve fitting*

Description

Fits parametric mortality curves to observed mortality rates using multiple optimisation strategies.

Usage

```
MC(
  x,
  m,
  curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
    "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  w = rep(1, length(x))
)
```

Arguments

x	vector of ages.
m	vector of mortality rates.
curve	name of mortality curve (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
w	vector of weights (default = 1).

Details

User can choose one of the following 14 mortality curves (with their suitable age ranges in brackets):

- Gompertz (1825) (ages 30-90) - $m_x = Be^{Cx}$
- Makeham (1860) (ages 20-90) - $m_x = A + Be^{Cx}$
- Oppermann (1870) (ages 0-20) - $m_x = \frac{A}{\sqrt{x+1}} + B + C\sqrt{x+1}$
- Thiele (1871) (ages 0-90) - $m_x = A_1e^{-B_1x} + A_2e^{-0.5B_2(x-C)^2} + A_3e^{B_3x}$
- Wittstein & Bumsted (1883) (ages 0-90) - $m_x = \frac{A^{-(Bx)^N}}{B} + A^{-(M-x)^N}$

- Perks (1932) (ages 20-100+) - $m_x = \frac{A+BC^x}{1+DC^x}$
- Weibull (1939) (ages 50-100+) - $m_x = Bx^C$
- Van der Maen (1943) (ages 80-100+) - $m_x = A + Bx + Cx^2 + \frac{I}{N-x}$
- Beard (1971) (ages 50-100+) - $m_x = \frac{Ae^{Cx}}{1+Be^{Cx}}$
- Heligman & Pollard (1980) (ages 0-100+) - $m_x = A^{(x+B)^C} + De^{-E(ln(x)-ln(F))^2} + \frac{GH^x}{1+GH^x}$
- Rogers & Planck (1983) (ages 0-100+) - $m_x = A_0 + A_1e^{-Ax} + A_2e^{-B(x-U)-e^{-C(x-U)}} + A_3e^{Dx}$
- Siler (1983) (ages 0-90) - $m_x = A_1e^{-B_1x} + A_2 + A_3e^{B_3x}$
- Martinelle (1987) (ages 20-100+) - $m_x = \frac{A+Be^{Cx}}{1+De^{Cx}} + Ee^{Cx}$
- Thatcher (1999) (ages 20-100+) - $m_x = A + \frac{Be^{Cx}}{1+De^{Cx}}$

The mortality curves are fitted by employing multiple optimisation strategies simultaneously (including PORT routines, Nelder-Mead method, and Levenberg-Marquardt algorithm) and selecting one with smallest weighted least squares on $ln(m_x)$. Constrained parameterisations offer traditional interpretability while unconstrained parameterisations provide increased flexibility.

Value

An object of class based on selected mortality curve with associated S3 methods coef, fitted, predict, plot, deviance, and residuals.

References

Gompertz, B. (1825). On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. Philosophical Transactions of the Royal Society of London, 115(1825), 513-583.

Makeham, W.M. (1860). On the law of mortality and the construction of annuity tables. Journal of the Institute of Actuaries, 8(6), 301-310.

Oppermann, L.H.F. (1870). On the graduation of life tables, with special application to the rate of mortality in infancy and childhood. The Institute of Actuaries.

Thiele, T.N. (1871). On a mathematical formula to express the rate of mortality throughout the whole of life, tested by a series of observations made use of by the Danish Life Insurance Company of 1871. Journal of the Institute of Actuaries and Assurance Magazine, 16(5), 313-329.

Wittstein, T. and Bumsted, D.A. (1883). The mathematical law of mortality. Journal of the Institute of Actuaries and Assurance Magazine, 24(3), 153-173.

Perks, W. (1932). On some experiments in the graduation of mortality statistics. Journal of the Institute of Actuaries, 63(1), 12-57.

Weibull, W. (1951). A statistical distribution function of wide applicability. Journal of Applied Mechanics, 18(3), 293-297.

Beard, R.E. (1971). Some aspects of theories of mortality, cause of death analysis, forecasting and stochastic processes. Biological Aspects of Demography, 57-68.

Heligman, L. and Pollard, J.H. (1980). The age pattern of mortality. Journal of the Institute of Actuaries, 107(1), 49-80.

Rogers, A. and Planck, F. (1983). Model: A general program for estimating parametrized model schedules of fertility, mortality, migration, and marital and labor force status transitions. IIASA Working Paper WP-83-102.

Siler, W. (1983). Parameters of mortality in human populations with widely varying life spans. Statistics in Medicine, 2(3), 373-380.

Martinelle, S. (1987). A generalized Perks formula for old-age mortality. Statistiska Centralbyran.

Thatcher, A.R. (1999). The long-term pattern of adult mortality and the highest attained age. Journal of the Royal Statistical Society Series A, 162(1), 5-43.

Tabeau, E. (2001). A review of demographic forecasting models for mortality. Forecasting Mortality in Developed Countries. European Studies of Population, Vol 9.

Examples

```
x <- 60:89
set.seed(123); m <- 0.0000082*exp(0.10771*c(60:89)+rnorm(30,0,0.1))
fit <- MC(x=x,m=m,curve="gompertz")
coef(fit)
fitted(fit)
predict(fit,62.5)
plot(fit)
deviance(fit)
residuals(fit)
```

---

modelS3 *Common S3 methods for stochastic mortality models*

Description

Common S3 methods for objects returned by `LCS()`, `RHS()`, `APCS()`, `CBDS()`, `CBDCS()`, `CBDQCS()`, and `STARS()`:

coef(object) gives estimated parameter values.

forecast::forecast(object) gives mortality forecasts smoothed by selected mortality curve.

plot(object) plots estimated parameter values, standardised residuals heatmap, data, and mortality forecasts smoothed by selected mortality curve.

residuals(object) gives standardised residuals on $ln(m_{x,t})$.

RHS *Renshaw-Haberman model*

Description

Fits and forecasts mortality rates using Renshaw-Haberman model.

Usage

```
RHS(
  x,
  M,
  curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
    "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10,
  jumpoff = 1
)
```

Arguments

`x`	vector of ages.
`M`	matrix of mortality rates (rows as years and columns as ages).
`curve`	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
`h`	forecast horizon (default = 10).
`jumpoff`	if 1, forecasts are based on estimated parameters only; if 2, forecasts are anchored to observed mortality rates in final year (default = 1).

Details

The Renshaw-Haberman (RH) model is specified as

$ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \gamma_{t-x} + \epsilon_{x,t}.$

The model is estimated by Newton updating scheme and is forecasted by ARIMA applied to κ_t and γ_c. Constraints include sum of β_x is one, sum of κ_t is zero, and sum of γ_c is zero. It can be applied to whole age range.

Value

An object of class RHS with associated S3 methods coef, forecast, plot, and residuals.

References

Haberman, S. and Renshaw, A. (2011). A comparative study of parametric mortality projection models. Insurance: Mathematics and Economics, 48(1), 35-55.

Examples

```
x <- 60:89
a <- c(-4.8499,-4.7676,-4.6719,-4.5722,-4.4847,-4.3841,-4.2813,-4.1863,-4.0861,-3.9962,
-3.8885,-3.7896,-3.6853,-3.5737,-3.4728,-3.3718,-3.2586,-3.1474,-3.0371,-2.9206,
-2.7998,-2.6845,-2.5653,-2.4581,-2.3367,-2.2159,-2.1017,-1.9941,-1.8821, -1.7697)
b <- c(0.0283,0.0321,0.0335,0.0336,0.0341,0.0358,0.0368,0.0403,0.0392,0.0395,
0.0396,0.0399,0.0397,0.0386,0.039,0.0375,0.0367,0.0368,0.035,0.0354,
0.0336,0.0323,0.0313,0.0295,0.0282,0.0265,0.024,0.0226,0.0219,0.0183)
k <- c(12.11,10.69,11.18,9.64,9.35,8.21,6.89,5.74,4.56,3.6,
3.27,2.04,1.11,-0.44,-1.05,-1.03,-1.84,-2.9,-4.03,-4.12,
-5.18,-5.64,-6,-6.51,-6.91,-6.9,-8.32,-8.53,-9.69,-9.31)
set.seed(123)
M <- exp(outer(k,b)+matrix(a,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.035))
fit <- RHS(x=x,M=M,curve="makeham",h=30,jumpoff=2)
coef(fit)
forecast::forecast(fit)
plot(fit)
residuals(fit)
```

STARS *Spatial-temporal autoregressive model*

Description

Fits and forecasts mortality rates using spatial-temporal autoregressive model.

Usage

```
STARS(
  x,
  M,
  curve = c("gompertz", "makeham", "oppermann", "thiele", "wittsteinbumsted", "perks",
    "weibull", "vandermaen", "beard", "heligmanpollard", "rogersplanck", "siler",
    "martinelle", "thatcher", "gompertz2", "makeham2", "oppermann2", "thiele2",
    "wittsteinbumsted2", "perks2", "weibull2", "vandermaen2", "beard2",
    "heligmanpollard2", "rogersplanck2", "siler2", "martinelle2", "thatcher2"),
  h = 10
)
```

Arguments

x	vector of ages.
M	matrix of mortality rates (rows as years and columns as ages).
curve	name of mortality curve for smoothing forecasted mortality rates (including gompertz, makeham, oppermann, thiele, wittsteinbumsted, perks, weibull, vandermaen, beard, heligmanpollard, rogersplanck, siler, martinelle, thatcher, gompertz2, makeham2, oppermann2, thiele2, wittsteinbumsted2, perks2, weibull2, vandermaen2, beard2, heligmanpollard2, rogersplanck2, siler2, martinelle2, thatcher2, where first 14 curves' parameters are unconstrained and last 14 curves' parameters are generally restricted to be positive).
h	forecast horizon (default = 10).

Details

The spatial-temporal autoregressive (STAR) model is specified as

$ln(M_t) = \mu + Rln(M_{t-1}) + E_t,$

where μ contains intercept parameters and R is banded matrix with non-zero entries in main diagonal and two subdiagonals below that.

The model is estimated by constrained regression and is forecasted via its autoregressive nature. Constraints in R include: all non-zero entries are between zero and one and sum of each row is one. It can be applied to whole age range.

Value

An object of class STARS with associated S3 methods coef, forecast, plot, and residuals.

References

Li, H. and Lu, Y. (2017). Coherent forecasting of mortality rates: A sparse vector-autoregression approach. ASTIN Bulletin, 47(2), 563-600.

Examples

```
x <- 60:89
a <- c(-4.8499,-4.7676,-4.6719,-4.5722,-4.4847,-4.3841,-4.2813,-4.1863,-4.0861,-3.9962,
-3.8885,-3.7896,-3.6853,-3.5737,-3.4728,-3.3718,-3.2586,-3.1474,-3.0371,-2.9206,
-2.7998,-2.6845,-2.5653,-2.4581,-2.3367,-2.2159,-2.1017,-1.9941,-1.8821, -1.7697)
b <- c(0.0283,0.0321,0.0335,0.0336,0.0341,0.0358,0.0368,0.0403,0.0392,0.0395,
0.0396,0.0399,0.0397,0.0386,0.039,0.0375,0.0367,0.0368,0.035,0.0354,
0.0336,0.0323,0.0313,0.0295,0.0282,0.0265,0.024,0.0226,0.0219,0.0183)
k <- c(12.11,10.69,11.18,9.64,9.35,8.21,6.89,5.74,4.56,3.6,
3.27,2.04,1.11,-0.44,-1.05,-1.03,-1.84,-2.9,-4.03,-4.12,
-5.18,-5.64,-6,-6.51,-6.91,-6.9,-8.32,-8.53,-9.69,-9.31)
set.seed(123)
M <- exp(outer(k,b)+matrix(a,nrow=30,ncol=30,byrow=TRUE)+rnorm(900,0,0.035))
fit <- STARS(x=x,M=M,curve="makeham",h=30)
coef(fit)
forecast::forecast(fit)
```

```
plot(fit)
residuals(fit)
```

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