Package ‘mstudentd’

July 9, 2024

Type Package

Title Multivariate t Distribution

Version 1.1.1

Maintainer Pierre Santagostini <pierre.santagostini@agrocampus-ouest.fr>

Description Distance between multivariate t distributions, as presented by N. Bouhlel and D. Rousseau (2023) <doi:10.1109/LSP.2023.3324594>.

Depends R (>= 3.3.0)

Imports rgl, MASS, data.table

License GPL (>= 3)

URL https://forgemia.inra.fr/imhorphen/mstudentd

BugReports https://forgemia.inra.fr/imhorphen/mstudentd/-/issues

Encoding UTF-8

RoxygenNote 7.3.2

Suggests testthat (>= 3.2.1)

Config/testthat/edition 3

NeedsCompilation no

Author Pierre Santagostini [aut, cre],
Nizar Bouhlel [aut]

Repository CRAN

Date/Publication 2024-07-09 17:00:02 UTC

Contents

`mstudentd-package` *Tools for Multivariate t Distributions*

Description

This package provides tools for multivariate t distributions (MTD):

- Calculation of distances/divergences between MTD:
  - Renyi divergence, Bhattacharyya distance, Hellinger distance: `diststudent`
  - Kullback-Leibler divergence: `kldstudent`
- Tools for MTD:
  - Probability density: `dmtd`
  - Simulation from a MTD: `rmtd`
  - Plot of the density of a MTD with 2 variables: `plotmtd`, `contourmtd`

Author(s)

Pierre Santagostini pierre.santagostini@agrocampus-ouest.fr, Nizar Bouhlel nizar.bouhlel@agrocampus-ouest.fr

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions, IEEE Signal Processing Letters. doi:10.1109/LSP.2023.3324594 #' @keywords internal

See Also

Useful links:

- `https://forgemia.inra.fr/imhorphen/mstudentd`
- Report bugs at `https://forgemia.inra.fr/imhorphen/mstudentd/-/issues`

contourmtd *Contour Plot of the Bivariate t Density*

Description

Draws the contour plot of the probability density of the multivariate t distribution with 2 variables with location parameter `mu` and scatter matrix `Sigma`.

Usage

```
contourmtd(nu, mu, Sigma,
                   xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                   ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]),
                   zlim = NULL, npt = 30, nx = npt, ny = npt,
                   main = "Multivariate t density",
                   sub = NULL, nlevels = 10,
                   levels = pretty(zlim, nlevels), tol = 1e-6, ...)
```

Arguments

`nu`	numeric. The degrees of freedom.
`mu`	length 2 numeric vector.
`Sigma`	symmetric, positive-definite square matrix of order 2. The scatter matrix.
`xlim, ylim`	x-and y- limits.
`zlim`	z- limits. If NULL, it is the range of the values of the density on the x and y values within `xlim` and `ylim`.
`npt`	number of points for the discretisation.
`nx, ny`	number of points for the discretisation among the x- and y- axes.
`main, sub`	main and sub title, as for `title`.
`nlevels, levels`	arguments to be passed to the `contour` function.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. see `dmtd`.
`...`	additional arguments to `plot.window`, `title`, `Axis` and `box`, typically graphical parameters such as `cex.axis`.

Value

Returns invisibly the probability density function.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

dmtd: probability density of a multivariate t density

plotmtd: 3D plot of a bivariate t density.

Examples

```
nu <- 1
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
contourmtd(nu, mu, Sigma)
```

---

diststudent *Distance/Divergence between Centered Multivariate t Distributions*

Description

Computes the distance or divergence (Renyi divergence, Bhattacharyya distance or Hellinger distance) between two random vectors distributed according to multivariate t distributions (MTD) with zero mean vector.

Usage

```
diststudent(nu1, Sigma1, nu2, Sigma2,
                    dist = c("renyi", "bhattacharyya", "hellinger"),
                    bet = NULL, eps = 1e-06)
```

Arguments

nu1	numéric. The degrees of freedom of the first distribution.
Sigma1	symmetric, positive-definite matrix. The correlation matrix of the first distribution.
nu2	numéric. The degrees of freedom of the second distribution.
Sigma2	symmetric, positive-definite matrix. The correlation matrix of the second distribution.
dist	character. The distance or divergence used. One of "renyi" (default), "battacharyya" or "hellinger".
bet	numeric, positive and not equal to 1. Order of the Renyi divergence. Ignored if distance="bhattacharyya" or distance="hellinger".
eps	numeric. Precision for the computation of the partial derivative of the Lauricella D-hypergeometric function (see Details). Default: 1e-06.

Details

Given X_1, a random vector of R^p distributed according to the MTD with parameters $(\nu_1, \mathbf{0}, \Sigma_1)$ and X_2, a random vector of R^p distributed according to the MTD with parameters $(\nu_2, \mathbf{0}, \Sigma_2)$.

Let $\delta_1 = \frac{\nu_1+p}{2}\beta$, $\delta_2 = \frac{\nu_2+p}{2}(1-\beta)$ and $\lambda_1, \dots, \lambda_p$ the eigenvalues of the square matrix $\Sigma_1\Sigma_2^{-1}$ sorted in increasing order:

$$\lambda_1 < \dots < \lambda_{p-1} < \lambda_p$$

The Renyi divergence between X_1 and X_2 is:

$$D_R^\beta(\mathbf{X}_1||\mathbf{X}_1) = \frac{1}{\beta-1}\Bigg[\beta \ln\left(\frac{\Gamma\left(\frac{\nu_1+p}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\nu_2^{\frac{p}{2}}}{\Gamma\left(\frac{\nu_2+p}{2}\right)\Gamma\left(\frac{\nu_1}{2}\right)\nu_1^{\frac{p}{2}}}\right) + \ln\left(\frac{\Gamma\left(\frac{\nu_2+p}{2}\right)}{\Gamma\left(\frac{\nu_2}{2}\right)}\right) + \ln\left(\frac{\Gamma\left(\delta_1+\delta_2-\frac{p}{2}\right)}{\Gamma(\delta_1+\delta_2)}\right) - \frac{\beta}{2}\sum_{i=1}^p \ln\lambda_i + \ln F_D\Bigg]$$

with F_D given by:

- If $\frac{\nu_1}{\nu_2}\lambda_1 > 1$: $F_D = F_D^{(p)}\left(\delta_1, \underbrace{\frac{1}{2},\dots,\frac{1}{2}}_{p}; \delta_1+\delta_2; 1-\frac{\nu_2}{\nu_1\lambda_1}, \dots, 1-\frac{\nu_2}{\nu_1\lambda_p}\right)$
- If $\frac{\nu_1}{\nu_2}\lambda_p < 1$: $F_D = \prod_{i=1}^p\left(\frac{\nu_1}{\nu_2}\lambda_i\right)^{\frac{1}{2}} F_D^{(p)}\left(\delta_2, \underbrace{\frac{1}{2},\dots,\frac{1}{2}}_{p}; \delta_1+\delta_2; 1-\frac{\nu_1}{\nu_2}\lambda_1, \dots, 1-\frac{\nu_1}{\nu_2}\lambda_p\right)$
- If $\frac{\nu_1}{\nu_2}\lambda_1 < 1$ and $\frac{\nu_1}{\nu_2}\lambda_p > 1$:
  $$F_D = \left(\frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)^{\delta_2}\prod_{i=1}^p\left(\frac{\nu_1}{\nu_2}\lambda_i\right)^{\frac{1}{2}} F_D^{(p)}\left(\delta_2, \underbrace{\frac{1}{2},\dots,\frac{1}{2}}_{p}, \delta_1+\delta_2-\frac{p}{2}; \delta_1+\delta 2; 1-\frac{\lambda_1}{\lambda_p}, \dots, 1-\frac{\lambda_{p-1}}{\lambda_p}, 1-\frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)$$

where $F_D^{(p)}$ is the Lauricella D-hypergeometric function defined for p variables:

$$F_D^{(p)}\left(a; b_1, ..., b_p; g; x_1, ..., x_p\right) = \sum_{m_1 \geq 0} ... \sum_{m_p \geq 0} \frac{(a)_{m_1+...+m_p}(b_1)_{m_1}...(b_p)_{m_p}}{(g)_{m_1+...+m_p}}\frac{x_1^{m_1}}{m_1!}...\frac{x_p^{m_p}}{m_p!}$$

Its computation uses the `lauricella` function.

The Bhattacharyya distance is given by:

$$D_B(\mathbf{X}_1||\mathbf{X}_2) = \frac{1}{2}D_R^{1/2}(\mathbf{X}_1||\mathbf{X}_2)$$

And the Hellinger distance is given by:

$$D_H(\mathbf{X}_1||\mathbf{X}_2) = 1 - \exp\left(-\frac{1}{2}D_R^{1/2}(\mathbf{X}_1||\mathbf{X}_2)\right)$$

Value

A numeric value: the Renyi divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the result of the Lauricella D-hypergeometric function,see Details) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions, IEEE Signal Processing Letters. doi:10.1109/LSP.2023.3324594

Examples

```
nu1 <- 2
Sigma1 <- matrix(c(2, 1.2, 0.4, 1.2, 2, 0.6, 0.4, 0.6, 2), nrow = 3)
nu2 <- 4
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)

# Renyi divergence
diststudent(nu1, Sigma1, nu2, Sigma2, bet = 0.25)
diststudent(nu2, Sigma2, nu1, Sigma1, bet = 0.25)

# Bhattacharyya distance
diststudent(nu1, Sigma1, nu2, Sigma2, dist = "bhattacharyya")
diststudent(nu2, Sigma2, nu1, Sigma1, dist = "bhattacharyya")

# Hellinger distance
diststudent(nu1, Sigma1, nu2, Sigma2, dist = "hellinger")
diststudent(nu2, Sigma2, nu1, Sigma1, dist = "hellinger")
```

---

dmtd — *Density of a Multivariate t Distribution*

Description

Density of the multivariate (p variables) t distribution (MTD) with degrees of freedom `nu`, mean vector `mu` and correlation matrix `Sigma`.

Usage

```
dmtd(x, nu, mu, Sigma, tol = 1e-6)
```

Arguments

`x`	length p numeric vector.
`nu`	numeric. The degrees of freedom.
`mu`	length p numeric vector. The mean vector.
`Sigma`	symmetric, positive-definite square matrix of order p. The correlation matrix.
`tol`	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma.

Details

The density function of a multivariate t distribution with p variables is given by:

$$f(\mathbf{x}|\nu, \boldsymbol{\mu}, \Sigma) = \frac{\Gamma\left(\frac{\nu+p}{2}\right)|\Sigma|^{-1/2}}{\Gamma\left(\frac{\nu}{2}\right)(\nu\pi)^{p/2}}\left(1 + \frac{1}{\nu}(\mathbf{x}-\boldsymbol{\mu})^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)^{-\frac{\nu+p}{2}}$$

When $p = 1$ (univariate case) it becomes:

$$f(x|\nu, \mu, \sigma^2) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}\sigma}\left(1 + \frac{(x-\mu)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}$$

Value

The value of the density.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

Examples

```
nu <- 1
mu <- c(0, 1, 4)
Sigma <- matrix(c(0.8, 0.3, 0.2, 0.3, 0.2, 0.1, 0.2, 0.1, 0.2), nrow = 3)
dmtd(c(0, 1, 4), nu, mu, Sigma)
dmtd(c(1, 2, 3), nu, mu, Sigma)

# Univariate
dmtd(1, 3, 0, 1)
dt(1, 3)
```

---

kldstudent — *Kullback-Leibler Divergence between Centered Multivariate t Distributions*

Description

Computes the Kullback-Leibler divergence between two random vectors distributed according to multivariate t distributions (MTD) with zero location vector.

Usage

```
kldstudent(nu1, Sigma1, nu2, Sigma2, eps = 1e-06)
```

Arguments

`nu1`	numeric. The degrees of freedom of the first distribution.
`Sigma1`	symmetric, positive-definite matrix. The scatter matrix of the first distribution.
`nu2`	numeric. The degrees of freedom of the second distribution.
`Sigma2`	symmetric, positive-definite matrix. The scatter matrix of the second distribution.
`eps`	numeric. Precision for the computation of the partial derivative of the Lauricella D-hypergeometric function (see Details). Default: 1e-06.

Details

Given X_1, a random vector of $\mathbb{R}^p$ distributed according to the centered MTD with parameters $(\nu_1, 0, \Sigma_1)$ and X_2, a random vector of $\mathbb{R}^p$ distributed according to the MCD with parameters $(\nu_2, 0, \Sigma_2)$.

Let $\lambda_1, \dots, \lambda_p$ the eigenvalues of the square matrix $\Sigma_1 \Sigma_2^{-1}$ sorted in increasing order:

$$\lambda_1 < \dots < \lambda_{p-1} < \lambda_p$$

The Kullback-Leibler divergence of X_1 from X_2 is given by:

$$D_{KL}(\mathbf{X}_1 \| \mathbf{X}_2) = \ln\left(\frac{\Gamma\left(\frac{\nu_1+p}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)\nu_2^{\frac{p}{2}}}{\Gamma\left(\frac{\nu_2+p}{2}\right)\Gamma\left(\frac{\nu_1}{2}\right)\nu_1^{\frac{p}{2}}}\right) + \frac{\nu_2-\nu_1}{2}\left[\psi\left(\frac{\nu_1+p}{2}\right) - \psi\left(\frac{\nu_1}{2}\right)\right] - \frac{1}{2}\sum_{i=1}^{p}\ln\lambda_i - \frac{\nu_2+p}{2} \times D$$

where ψ is the digamma function (see Special) and D is given by:

- If $\frac{\nu_1}{\nu_2}\lambda_1 > 1$:

$$D = \prod_{i=1}^{p}\left(\frac{\nu_2}{\nu_1}\frac{1}{\lambda_i}\right)^{\frac{1}{2}} \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(\frac{\nu_1+p}{2}, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{p}; a + \frac{\nu_1+p}{2}; 1 - \frac{\nu_2}{\nu_1}\frac{1}{\lambda_1}, \dots, 1 - \frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)\right\}\Bigg|_{a=0}$$

- If $\frac{\nu_1}{\nu_2}\lambda_p < 1$:

$$D = \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(a, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{p}; a + \frac{\nu_1+p}{2}; 1 - \frac{\nu_1}{\nu_2}\lambda_1, \dots, 1 - \frac{\nu_1}{\nu_2}\lambda_p\right)\right\}\Bigg|_{a=0}$$

- If $\frac{\nu_1}{\nu_2}\lambda_1 < 1$ and $\frac{\nu_1}{\nu_2}\lambda_p > 1$:

$$D = -\ln\left(\frac{\nu_1}{\nu_2}\lambda_p\right) + \frac{\partial}{\partial a}\left\{F_D^{(p)}\left(a, \underbrace{\frac{1}{2}, \dots, \frac{1}{2}, a + \frac{\nu_1}{2}}_{p}; a + \frac{\nu_1+p}{2}; 1 - \frac{\lambda_1}{\lambda_p}, \dots, 1 - \frac{\lambda_{p-1}}{\lambda_p}, 1 - \frac{\nu_2}{\nu_1}\frac{1}{\lambda_p}\right)\right\}\Bigg|_{a=0}$$

$F_D^{(p)}$ is the Lauricella D-hypergeometric function defined for p variables:

$$F_D^{(p)}(a; b_1, ..., b_p; g; x_1, ..., x_p) = \sum_{m_1 \geq 0} ... \sum_{m_p \geq 0} \frac{(a)_{m_1+...+m_p}(b_1)_{m_1} ... (b_p)_{m_p}}{(g)_{m_1+...+m_p}} \frac{x_1^{m_1}}{m_1!} ... \frac{x_p^{m_p}}{m_p!}$$

Value

A numeric value: the Kullback-Leibler divergence between the two distributions, with two attributes `attr(, "epsilon")` (precision of the partial derivative of the Lauricella D-hypergeometric function,see Details) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel and D. Rousseau (2023), Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions, IEEE Signal Processing Letters. doi:10.1109/LSP.2023.3324594

Examples

```
nu1 <- 2
Sigma1 <- matrix(c(2, 1.2, 0.4, 1.2, 2, 0.6, 0.4, 0.6, 2), nrow = 3)
nu2 <- 4
Sigma2 <- matrix(c(1, 0.3, 0.1, 0.3, 1, 0.4, 0.1, 0.4, 1), nrow = 3)

kldstudent(nu1, Sigma1, nu2, Sigma2)
kldstudent(nu2, Sigma2, nu1, Sigma1)
```

---

lauricella — *Lauricella D-Hypergeometric Function*

Description

Computes the Lauricella D-hypergeometric Function function.

Usage

```
lauricella(a, b, g, x, eps = 1e-06)
```

Arguments

`a`	numeric.
`b`	numeric vector.
`g`	numeric.
`x`	numeric vector. `x` must have the same length as `b`.
`eps`	numeric. Precision for the nested sums (default 1e-06).

Details

If n is the length of the b and `x` vectors, the Lauricella D-hypergeometric Function function is given by:

$$F_D^{(n)}\left(a,b_1,...,b_n,g;x_1,...,x_n\right)=\sum_{m_1\geq 0}...\sum_{m_n\geq 0}\frac{(a)_{m_1+...+m_n}(b_1)_{m_1}...(b_n)_{m_n}}{(g)_{m_1+...+m_n}}\frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!}$$

where $(x)_p$ is the Pochhammer symbol (see `pochhammer`).

If $|x_i|<1, i=1,\dots,n$, this sum converges. Otherwise there is an error.

The `eps` argument gives the required precision for its computation. It is the `attr(, "epsilon")` attribute of the returned value.

Sometimes, the convergence is too slow and the required precision cannot be reached. If this happens, the `attr(, "epsilon")` attribute is the precision that was really reached.

Value

A numeric value: the value of the Lauricella function, with two attributes `attr(, "epsilon")` (precision of the result) and `attr(, "k")` (number of iterations).

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

N. Bouhlel and D. Rousseau, Exact Rényi and Kullback-Leibler Divergences Between Multivariate t-Distributions. IEEE Signal Processing Letters Processing Letters, vol. 26 no. 7, July 2019. doi:10.1109/LSP.2019.2915000

`lnpochhammer` *Logarithm of the Pochhammer Symbol*

Description

Computes the logarithm of the Pochhammer symbol.

Usage

```
lnpochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

So, if $n > 0$:

$$log\left((x)_n\right) = log(x) + log(x+1) + ... + log(x+n-1)$$

If $n = 0$, $log\left((x)_n\right) = log(1) = 0$

Value

Numeric value. The logarithm of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

See Also

pochhammer()

Examples

```
lnpochhammer(2, 0)
lnpochhammer(2, 1)
lnpochhammer(2, 3)
```

`plotmtd` *Plot of the Bivariate t Density*

Description

Plots the probability density of the multivariate t distribution with 2 variables with location parameter `mu` and scatter matrix `Sigma`.

Usage

```
plotmtd(nu, mu, Sigma, xlim = c(mu[1] + c(-10, 10)*Sigma[1, 1]),
                ylim = c(mu[2] + c(-10, 10)*Sigma[2, 2]), n = 101,
                xvals = NULL, yvals = NULL, xlab = "x", ylab = "y",
                zlab = "f(x,y)", col = "gray", tol = 1e-6, ...)
```

Arguments

nu	numeric. The degrees of freedom.
mu	length 2 numeric vector. The mean vector.
Sigma	symmetric, positive-definite square matrix of order 2. The correlation matrix.
xlim, ylim	x-and y- limits.
n	A one or two element vector giving the number of steps in the x and y grid, passed to plot3d.function.
xvals, yvals	The values at which to evaluate x and y. If used, xlim and/or ylim are ignored.
xlab, ylab, zlab	The axis labels.
col	The color to use for the plot. See plot3d.function.
tol	tolerance (relative to largest variance) for numerical lack of positive-definiteness in Sigma, for the estimation of the density. see dmtd.
...	Additional arguments to pass to plot3d.function.

Value

Returns invisibly the probability density function.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

dmtd: probability density of a multivariate t density

contourmtd: contour plot of a bivariate t density.

plot3d.function: plot a function of two variables.

Examples

```
nu <- 1
mu <- c(1, 4)
Sigma <- matrix(c(0.8, 0.2, 0.2, 0.2), nrow = 2)
plotmtd(nu, mu, Sigma)
```

`pochhammer` *Pochhammer Symbol*

Description

Computes the Pochhammer symbol.

Usage

```
pochhammer(x, n)
```

Arguments

`x`	numeric.
`n`	positive integer.

Details

The Pochhammer symbol is given by:

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)...(x+n-1)$$

Value

Numeric value. The value of the Pochhammer symbol.

Author(s)

Pierre Santagostini, Nizar Bouhlel

Examples

```
pochhammer(2, 0)
pochhammer(2, 1)
pochhammer(2, 3)
```

rmtd *Simulate from a Multivariate t Distribution*

Description

Produces one or more samples from the multivariate (p variables) t distribution (MTD) with degrees of freedom `nu`, mean vector `mu` and correlation matrix `Sigma`.

Usage

```
rmtd(n, nu, mu, Sigma, tol = 1e-6)
```

Arguments

`n`	integer. Number of observations.
`nu`	numeric. The degrees of freedom.
`mu`	length p numeric vector. The mean vector
`Sigma`	symmetric, positive-definite square matrix of order p. The correlation matrix.
`tol`	tolerance for numerical lack of positive-definiteness in Sigma (for `mvrnorm`, see Details).

Details

A sample from a MTD with parameters ν, $\boldsymbol{\mu}$ and Σ can be generated using:

$$X = \mu + \frac{Y}{\sqrt{\frac{u}{\nu}}}$$

where Y is a random vector distributed among a centered Gaussian density with covariance matrix Σ (generated using `mvrnorm`) and u is distributed among a Chi-squared distribution with ν degrees of freedom.

Value

A matrix with p columns and n rows.

Author(s)

Pierre Santagostini, Nizar Bouhlel

References

S. Kotz and Saralees Nadarajah (2004), Multivariate t Distributions and Their Applications, Cambridge University Press.

See Also

`dmtd`: probability density of a MTD.

Examples

```
nu <- 3
mu <- c(0, 1, 4)
Sigma <- matrix(c(1, 0.6, 0.2, 0.6, 1, 0.3, 0.2, 0.3, 1), nrow = 3)
x <- rmtd(10000, nu, mu, Sigma)
head(x)
dim(x)
mu; colMeans(x)
nu/(nu-2)*Sigma; var(x)
```

Index