

Solutions

Part	I	II	III	IV	V	VI	Total
Score							

Part I: Fill in the blanks. (6 questions; 3 points for each; 18 points in total.)

Notice: you MUST write the answers in the following tables.

Number	1	2	3
Answer			
Number	4	5	6
Answer			

- The first question $k > 0$, text $f(x) = \ln x - \frac{x}{e} + k$ text $(0, +\infty)$ text text text text text
text text text text text text text text text text text text text text text text text
text text text text text _____ 2.
- The second question $\vec{a} = (2, 1, 2), \vec{b} = (4, -1, 10), \vec{c} = \vec{b} - \lambda \vec{a}$, text text text $\vec{a} \perp \vec{c}$, text
text text text text text text text text text text text text text text $\lambda =$ _____ 3.
- The third question $\begin{vmatrix} 1 & 2 \\ -3 & x \end{vmatrix} = 0$, text text text text text text text text text text text
text text text text text text text $x =$ _____ -6.
- The fourth question $\alpha_1 = (1, 1, 0), \alpha_2 = (0, 1, 1), \alpha_3 = (1, 0, 1)$, text $\beta = (4, 5, 3)$ text
 $\alpha_1, \alpha_2, \alpha_3$ text text text text text text text text text text text text text text text text text
text text text text text text text text text text $\beta =$ _____ $3\alpha_1 + 2\alpha_2 + \alpha_3$.
- The fifth question ξ text text text $E\xi = 3, D\xi = 2$, text text text text text text text text text
text text text text text text text text $E\xi^2 =$ _____ 11.
- The sixth question ξ text text text text η text text text text $\xi \sim N(1, 4), \eta \sim N(2, 5)$,
text
text text text text text $\xi - 2\eta \sim$ _____ $N(-3, 24)$

Part II: Select one answer from four choices. (6 questions; 3 points for each; 18 points in total.)

Notice: you MUST write the answers in the following tables.

Number	1	2	3	4	5	6
Answer						

- 1.** The first question text, text text text text text text text text text text text text
text text text text text text text text text.....(C)

(A) first $\int f'(x) dx = f(x)$

(B) second $\int df(x) = f(x)$

(C) third $\frac{d}{dx} \left(\int f(x) dx \right) = f(x)$

(D) fourth $d \left(\int f(x) dx \right) = f(x)$

2. The second question $F(x)$ text $f(x)$ text, text..... (A)

(A) first choice $F(x)$ text $\Leftrightarrow f(x)$ text

(B) second choice $F(x)$ text text $\Leftrightarrow f(x)$ text

(C) third choice $F(x)$ text $\Leftrightarrow f(x)$ text text

(D) fourth choice $F(x)$ text text $\Leftrightarrow f(x)$ text text

3. The third question $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & x & 0 \\ 0 & 0 & 1 \end{pmatrix}$ text text $\lambda_1 = 1$ text $\lambda_2 = 2$, text $x = \dots \dots (\text{ B })$

(A) 2

(B) 1

(C) 0

(D) -1

- 4.** The fourth question $f = 4x_1^2 - 2x_1x_2 + 6x_2^2$ text text text text text text text text
text text text text text text text text text text text text (C)

(A) $\begin{pmatrix} 4 & -2 \\ -2 & 6 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 4 & -1 \\ -1 & 6 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$

5. The fifth question wrong text (B)

(A) first choice text text text text text text text text

(B) second choice text text text text text text text text text

(C) third choice text text text text text text text text

(D) fourth choice text text text text text text text text

- 6.** The sixth question X text (X_1, \dots, X_n) text wrong text text text text text text text
text text text text text text text text.....(D)

(A) text text text

(B) text n text

(C) X_1, \dots, X_n text

(D) $X_1 = X_2 = \cdots = X_n$

Part III: Work out math questions. (6 questions; 8 points for each; 48 points in total.)

1. The first question $\int e^{2x} (\tan x + 1)^2 dx$.

Solution. $I = \int e^{2x} \sec^2 x dx + 2 \int e^{2x} \tan x dx$ 2 points

$$= \int e^{2x} d(\tan x) + 2 \int e^{2x} \tan x dx$$
4 points
$$= e^{2x} \tan x - 2 \int e^{2x} \tan x dx + 2 \int e^{2x} \tan x dx$$
6 points
$$= e^{2x} \tan x + C$$
8 points

2. The second question $A(1, 2, -1), B(2, 3, 0), C(3, 3, 2)$ text $\triangle ABC$ text text text text text.

Solution. Text $\overrightarrow{AB} = (1, 1, 1), \overrightarrow{AC} = (2, 1, 3)$,2 points

$$\text{text } \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = (2, -1, -1),$$
4 points
$$\text{text } \triangle ABC \text{ text } S_{\triangle ABC} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \sqrt{6}.$$
6 points
Text text $2(x-2) - (y-3) - z = 0$, text $2x - y - z - 1 = 0$8 points

3. The third question $A = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$ text.

Solution. $A = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 0 & -1 & -6 & 1 \\ 0 & -6 & -8 & 2 \end{vmatrix} = 1 \cdot (-1)^{2+1} \begin{vmatrix} 1 & 2 & 3 \\ -1 & -6 & 1 \\ -6 & -8 & 2 \end{vmatrix} \dots\dots 4 \text{ points}$

$$= - \begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 4 \\ 0 & 4 & 20 \end{vmatrix} = - \begin{vmatrix} -4 & 4 \\ 4 & 20 \end{vmatrix} = -(-4 \cdot 20 - 4 \cdot 4) = 96 \dots\dots 8 \text{ points}$$

4. The fourth question, tex text $f = x_1^2 + 2x_1x_2 - 6x_1x_3 + 2x_2^2 - 12x_2x_3 + 9x_3^2$ text text
 $f = d_1y_1^2 + d_2y_2^2 + d_3y_3^2$.

Solution. $f = x_1^2 + 2x_1x_2 - 6x_1x_3 + 2x_2^2 - 12x_2x_3 + 9x_3^2$
 $= x_1^2 + 2x_1(x_2 - 3x_3) + (x_2 - 3x_3)^2 + x_2^2 - 6x_2x_3$
 $= (x_1 + x_2 - 3x_3)^2 + x_2^2 - 6x_2x_3 \dots\dots 3 \text{ points}$

$$= (x_1 + x_2 - 3x_3)^2 + x_2^2 - 2x_2 \cdot 3x_3 + (3x_3)^2 - 9x_3^2$$

$$= (x_1 + x_2 - 3x_3)^2 + (x_2 - 3x_3)^2 - 9x_3^2 \dots\dots 6 \text{ points}$$

Text $y_1 = x_1 + x_2 - 3x_3$, $y_2 = x_2 - 3x_3$, $y_3 = x_3$,

text $f = y_1^2 + y_2^2 - 9y_3^2$ text. $\dots\dots 8 \text{ points}$

5. The fifth question text text text 0.2 text text, text text 100 text text.

(1) text text text text text text ξ text 10 text 30 text.

(2) text text text text text text ξ text 10 text 30 text.

Solution. $E\xi = np = 100 \cdot 0.2 = 20$, $D\xi = npq = 100 \cdot 0.2 \cdot 0.8 = 16$2 points

$$(1) P(10 < \xi < 30) = P(|\xi - E\xi| < 10) \geq 1 - \frac{D\xi}{10^2} = 1 - \frac{16}{100} = 0.84. \text{4 points}$$

$$(2) P(10 < \xi < 30) \approx \Phi_0\left(\frac{30-20}{\sqrt{16}}\right) - \Phi_0\left(\frac{10-20}{\sqrt{16}}\right) \text{6 points}$$

$$= 2\Phi_0(2.5) - 1 = 2 \cdot 0.9938 - 1 = 0.9876 \text{8 points}$$

6. The sixth question $N(\mu, \sigma^2)$ text text 16 text, text text text 3160, text text 100. Text text $H_0: \mu = 3140$ text text ($\alpha = 0.01$).

Solution. (1) Text text $H_0: \mu = 3140$2 points

$$(2) \text{ Text text text } T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n-1). \text{3 points}$$

$$(3) \text{ Text text } t_\alpha = t_\alpha(n-1) = t_{0.01}(15) = 2.947. \text{5 points}$$

$$(4) \text{ Text text text } t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3160 - 3140}{100/4} = 0.8. \text{7 points}$$

$$(5) \text{ Text } |t| < t_\alpha, \text{ text text } H_0, \text{ text text text.} \text{8 points}$$

Part IV: Work out math proofs. (2 questions; 16 points in total.)

1. (9 points) The first question $\{x_n\}$ text $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2 + x_n}$. Text text text, text text text.

Proof. (1) Text, text $x_1 < 2$, text $x_k < 2$ text

$$x_{k+1} = \sqrt{2 + x_k} < \sqrt{2 + 2} = 2,$$

Text text text text text n text $x_n < 2$, text text text. Text text

$$\frac{x_{n+1}}{x_n} = \sqrt{\frac{2}{x_n^2} + \frac{1}{x_n}} > \sqrt{\frac{2}{2^2} + \frac{1}{2}} = 1,$$

Text text text text text. Text text text text text, Text text text text.4 points

(2) Text text text text A , text text text text text text

$$A = \sqrt{2 + A}.$$

Text text $A = 2$, text text $\{x_n\}$ text text text 2.

.....8 points

2. (7 points) The second question A text B text, text A text \bar{B} text.

Proof. $P(A \cdot \bar{B}) = P(A - B) = P(A - AB)$

.....2 points

$$= P(A) - P(AB) = P(A) - P(A)P(B)$$

.....4 points

$$= P(A)(1 - P(B)) = P(A)P(\bar{B})$$

.....6 points

Text text text A text text text \bar{B} text text text.

.....8 points

Appendix Some data may be used in the exam

$\Phi_0(0.5) = 0.6915$	$\Phi_0(1) = 0.8413$	$\Phi_0(2) = 0.9773$	$\Phi_0(2.5) = 0.9938$
$t_{0.01}(8) = 3.355$	$t_{0.01}(9) = 3.250$	$t_{0.01}(15) = 2.947$	$t_{0.01}(16) = 2.921$
$\chi_{0.005}^2(8) = 22.0$	$\chi_{0.005}^2(9) = 23.6$	$\chi_{0.005}^2(15) = 32.8$	$\chi_{0.005}^2(16) = 34.3$
$\chi_{0.995}^2(8) = 1.34$	$\chi_{0.995}^2(9) = 1.73$	$\chi_{0.995}^2(15) = 4.60$	$\chi_{0.995}^2(16) = 5.14$